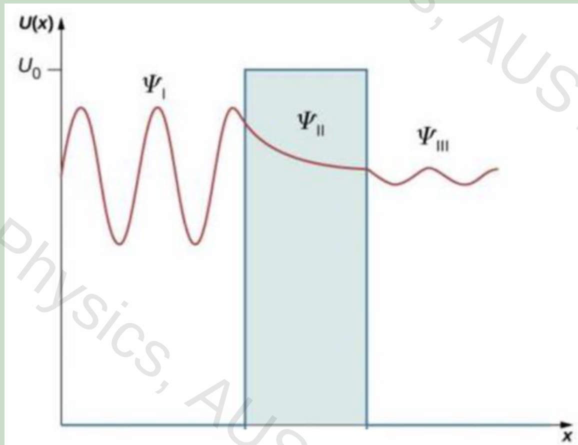
安徽理工大学
基础物理研究中心
成立大会暨学术报告会

量子隧穿和最小空间尺度的存在性



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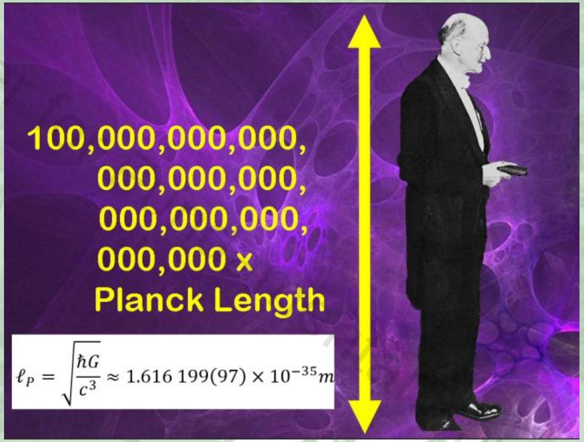

中国科学院固体物理研究所

2024年5月24日 @ 安徽理工大学

1927年索尔维会议

致敬量子力学的先贤们！

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SOLVAY CONFERENCE 1927

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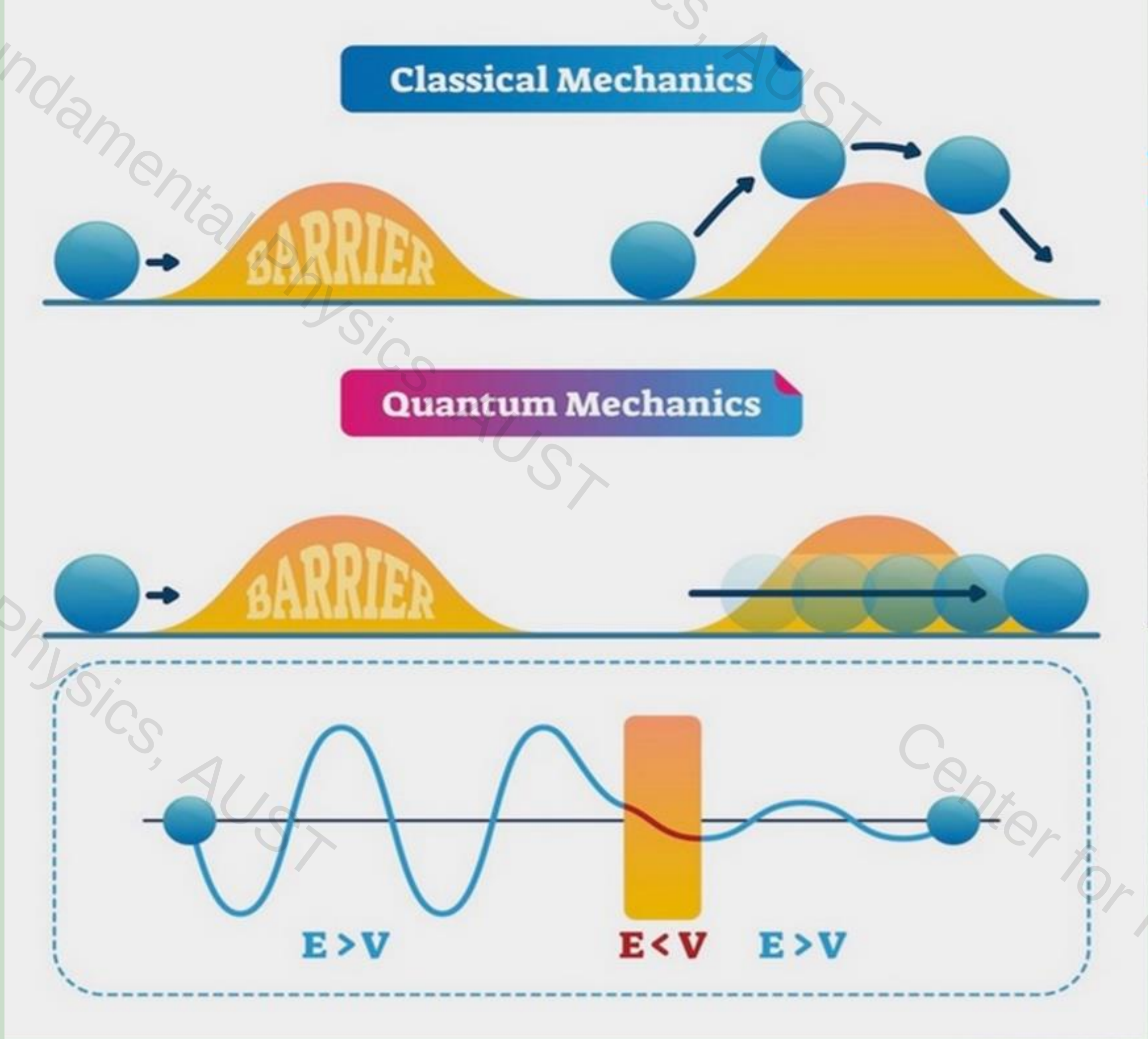
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

Absents : Sir W.H. BRAGG, H. DESLANDRES et E. VAN AUBEL

微观世界的崂山道士 — 量子隧穿现象



- 入射粒子的能量 (E) 低于势垒高度 (E_b) 所发生的势垒穿透现象
- 物理起源—**波粒二象性**

课题组的一些近期工作

- **Yong Yang**, Penetration of arbitrary double potential barriers with probability unity: Implications for testing the existence of a minimum length. ***Phys. Rev. Research* 6**, 013087 (2024).
- Y. W. Tong and **Yong Yang***, Hydrogen diffusion on graphene surface: the effects of neighboring adsorbate and quantum tunneling. ***J. Phys. Chem. C* 128**, 840 (2024).
- C. Bi and **Yong Yang***, Atomic Resonant Tunneling in the Surface Diffusion of H Atoms on Pt(111). ***J. Phys. Chem. C* 125**, 464 (2021).
- **Yong Yang*** and Y. Kawazoe, Adsorption and Diffusion of H Atoms on β-PtO2 Surface: The Role of Nuclear Quantum Effects. ***J. Phys. Chem. C* 123**, 13804 (2019).
- X. F. Yu$^{\#}$, Y. W. Tong$^{\#}$, **Yong Yang***, Activated dissociation of H2 on the Cu(001) surface: The role of quantum tunneling. ***Chinese Physics B* 32**, 108103 (2023). (Suggested by Editor)
- X. F. Yu, Y. W. Tong, **Yong Yang***, Quantum tunneling in the surface diffusion of single hydrogen atoms on Cu(001). ***Chinese Physics B* 32**, 086801 (2023). (Suggested by Editor)
- C. Bi, Q. Chen, W. Li and **Yong Yang***, Quantum nature of proton transferring across one-dimensional potential fields. ***Chinese Physics B* 30**, 046601 (2021). (Suggested by Editor)

微观粒子的波粒二象性

- **de Broglie Wavelength of a particle**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} \sim \frac{h}{\sqrt{2mk_BT}}$$

T **~ 300 K, the wavelength of electron, H, O, Pt atoms:**

λe ~ 76.3 Å → large wave characteristics expected!

Solving the wave equation—Schrödinger Equation.

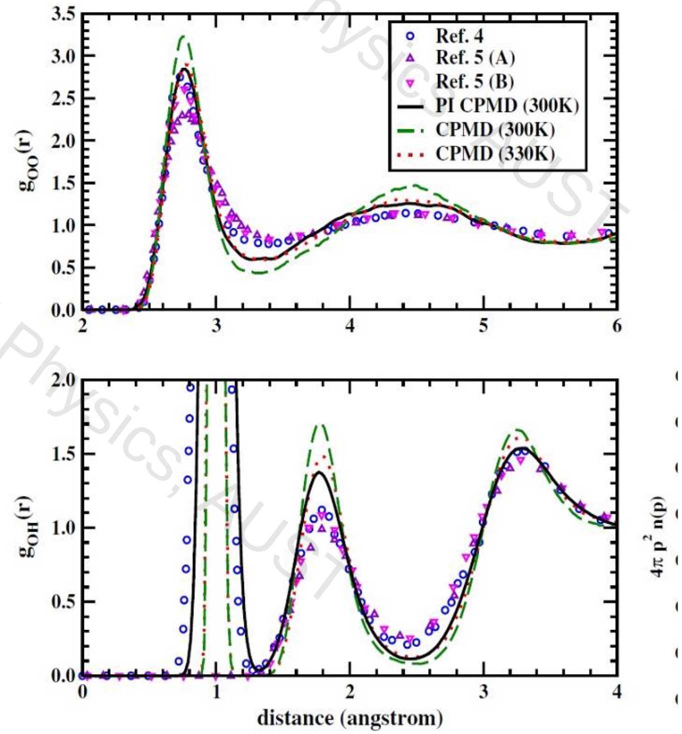
λ_H ~ 1.78 Å → comparable with bond lengths!

λ_O ~ 0.45 Å → smaller than atomic radii.

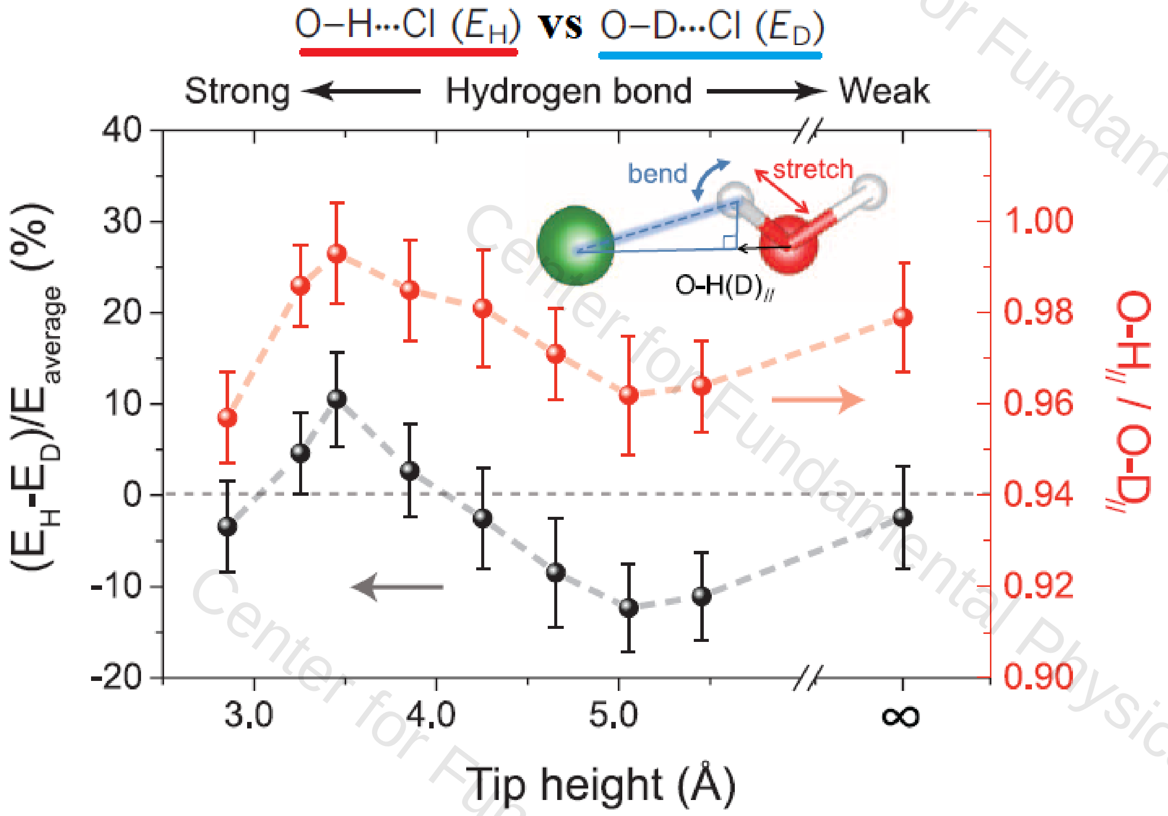
λ_{Pt} ~ 0.13 Å → much smaller than atomic radii.

波粒二象性的物理呈现

- 集体运动的元激发: 等离激元、声子、旋子
- 单粒子运动：衍射、干涉、隧穿/隧道效应
- 核量子效应 (凝聚



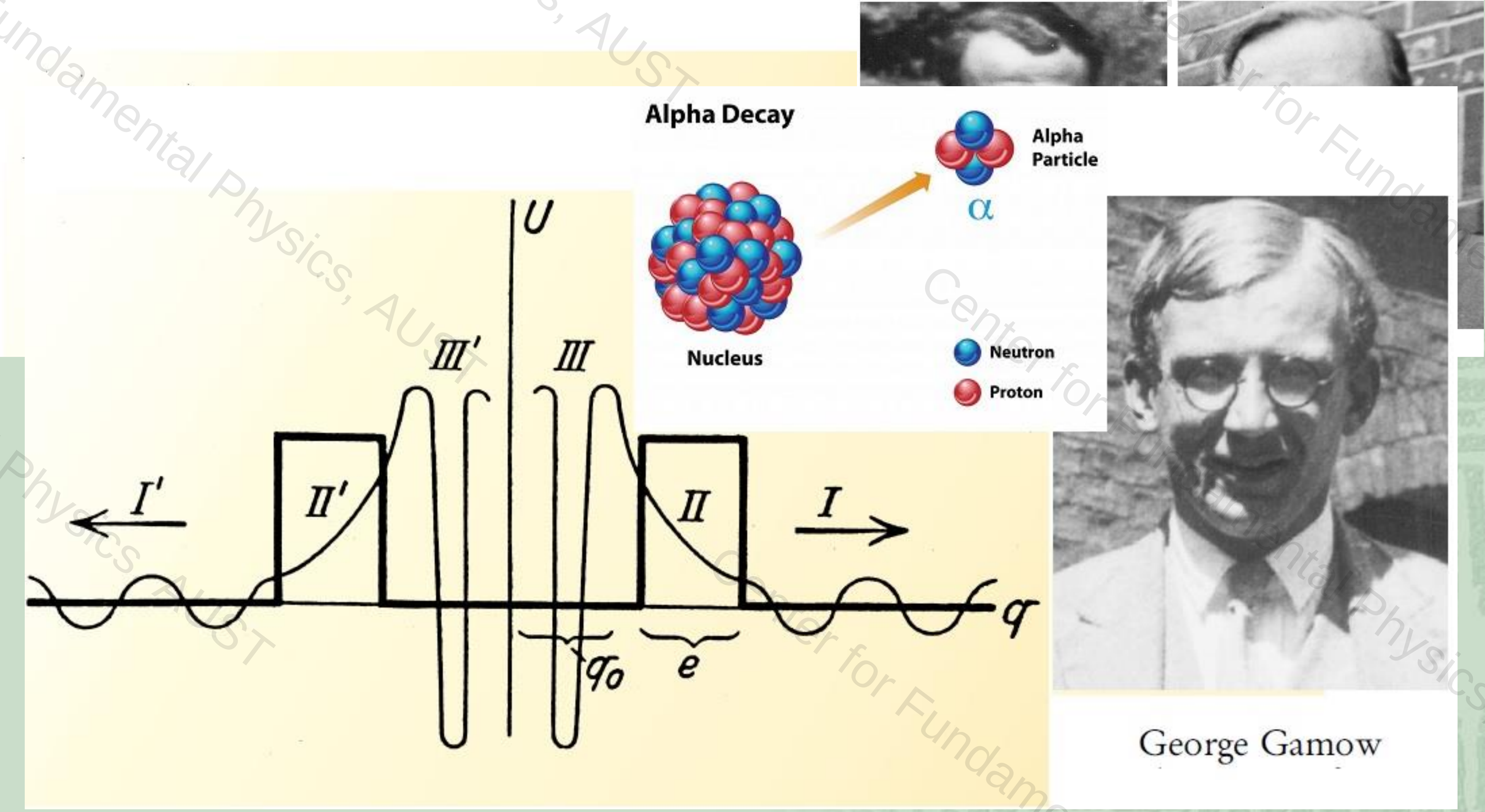
liquid water PRL **101**, 017801



Science 352, 321 (2016)

量子隧穿的历史回顾(1920~30s)

- 金属低温表面的电子场发射；**Alpha** 衰变



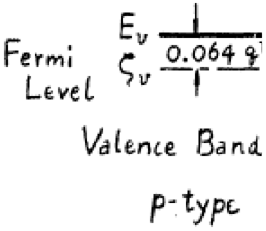
George Gamow

量子隧穿的历史回顾 (1950~60s)

New Phenomenon in N
***p*-n Junc**

Leo Esa
Tokyo Tsushin Kogyo, Limited,
(Received Octobe

Phys. Rev. 109, 603



Leo Esaki
江崎玲于奈

Volume 5, Number 4 PHYSICAL REVIEW LETTERS August 15, 1960

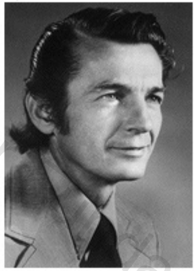
ENERGY GAP IN SUPERCONDUCTORS MEASURED BY ELECTRON TUNNELING

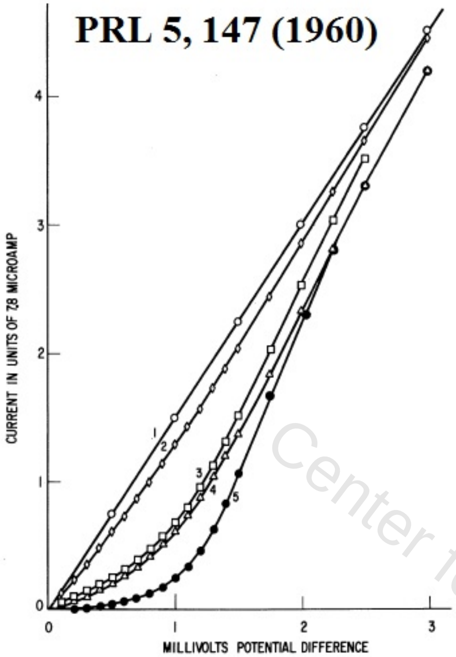
Ivar Giaever
General Electric Research Laboratory, Schenectady, New York
(Received July 5, 1960)

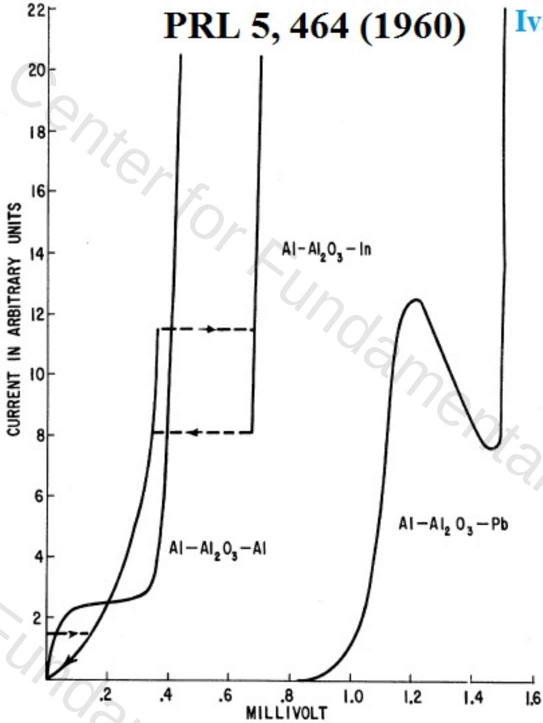
超导–绝缘体界面的单粒子隧道效应 ⟹ 超导能隙测量

ELECTRON TUNNELING BETWEEN TWO SUPERCONDUCTORS

Ivar Giaever
General Electric Research Laboratory, Schenectady, New York
(Received October 31, 1960)




PRL 5, 147 (1960)


PRL 5, 464 (1960)

Ivar Giaever

Nobel in Physics 1973

量子隧穿的历史回顾 (1970~80s)

Tunneling in a finite superlattice*

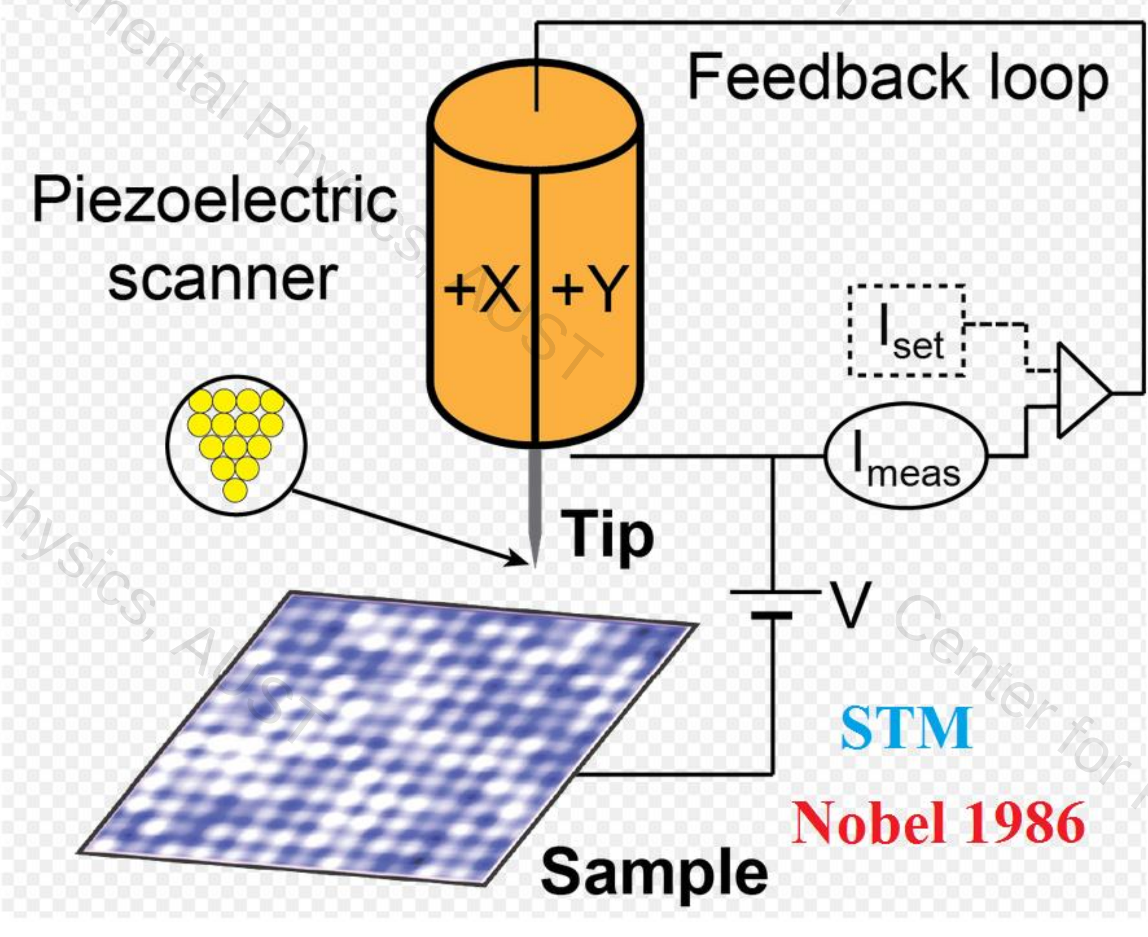
R. Tsu and L. Esaki   Appl. Phys. Lett. 22, 562 (1973)

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 9 March 1973)

半导体双势垒结构中的共振隧穿现象

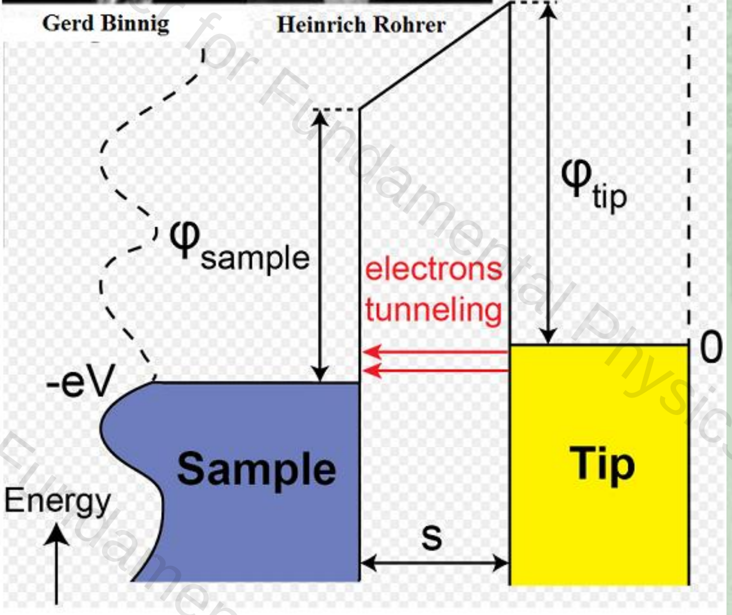
Resonant tunneling in semiconductor double barriers*



STM

Nobel 1986

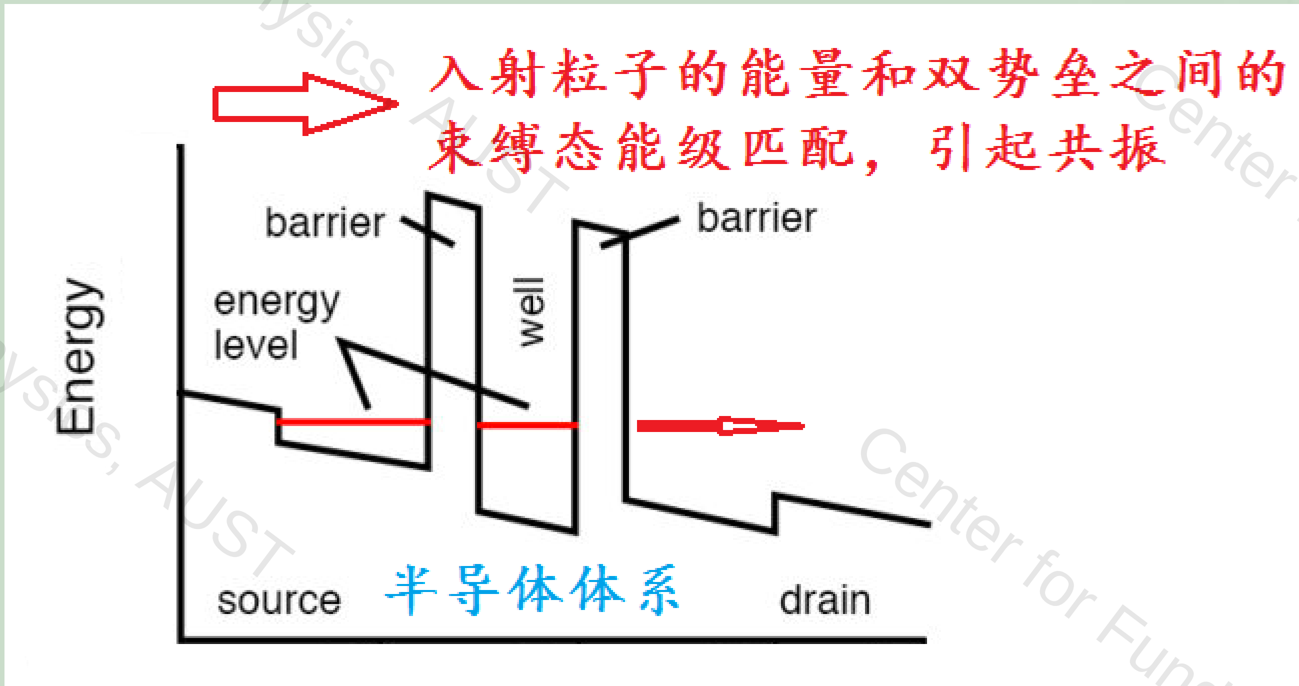
Gerd Binnig   **Heinrich Rohrer**



量子隧穿的特殊情形 — 共振隧穿

- 共振隧穿 (Resonant Tunneling, RT)

在特定条件下，入射粒子以100%的概率穿透遇到的势场（势垒、势阱）；物理模型—双势垒

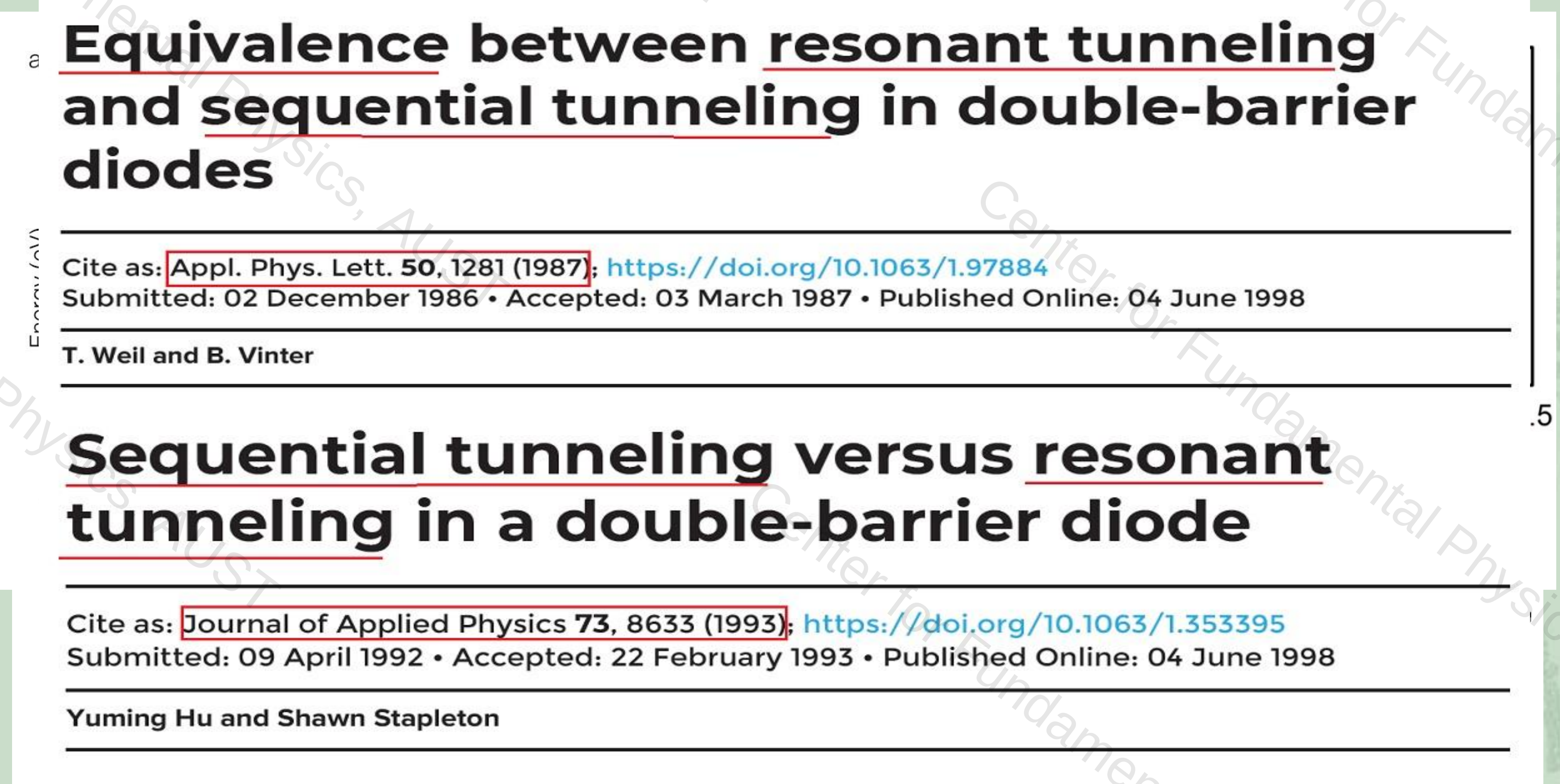


入射粒子的能量和双势垒之间的束缚态能级匹配，引起共振

透射物质波幅度的共振增强 → 100%透过

共振隧穿的实验研究

- 电子发生RT的可能实验证据—**I-V曲线的振荡行为**，以及对应的**负微分电阻**；机理尚存争议 (sequential tunneling, ST也能解释上述实验观测)；**原子分子的RT？尚未有实验观测！**



Equivalence between resonant tunneling and sequential tunneling in double-barrier diodes

Cite as: Appl. Phys. Lett. **50**, 1281 (1987); https://doi.org/10.1063/1.97884
Submitted: 02 December 1986 • Accepted: 03 March 1987 • Published Online: 04 June 1998

T. Weil and B. Vinter

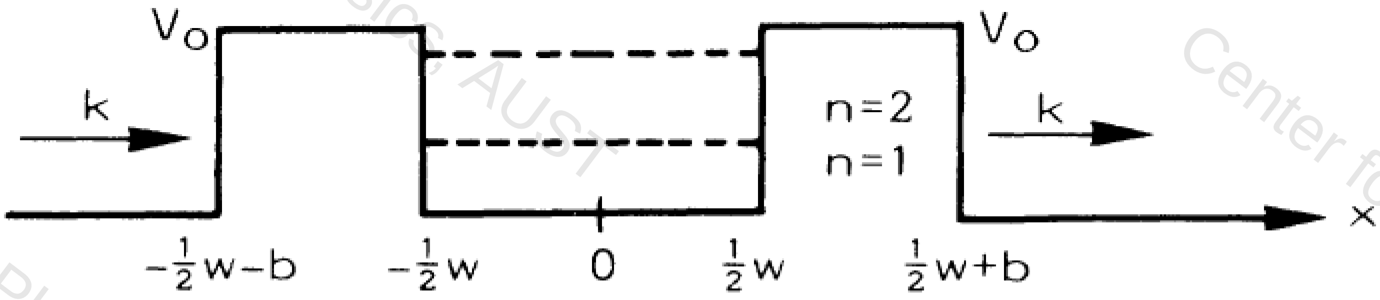
Sequential tunneling versus resonant tunneling in a double-barrier diode

Cite as: Journal of Applied Physics **73**, 8633 (1993); https://doi.org/10.1063/1.353395
Submitted: 09 April 1992 • Accepted: 22 February 1993 • Published Online: 04 June 1998

Yuming Hu and Shawn Stapleton

共振隧穿的理论研究

矩形双势垒



$$E=\frac{\hbar^2k^2}{2m},\quad V_0=\frac{\hbar^2k_0^2}{2m},$$

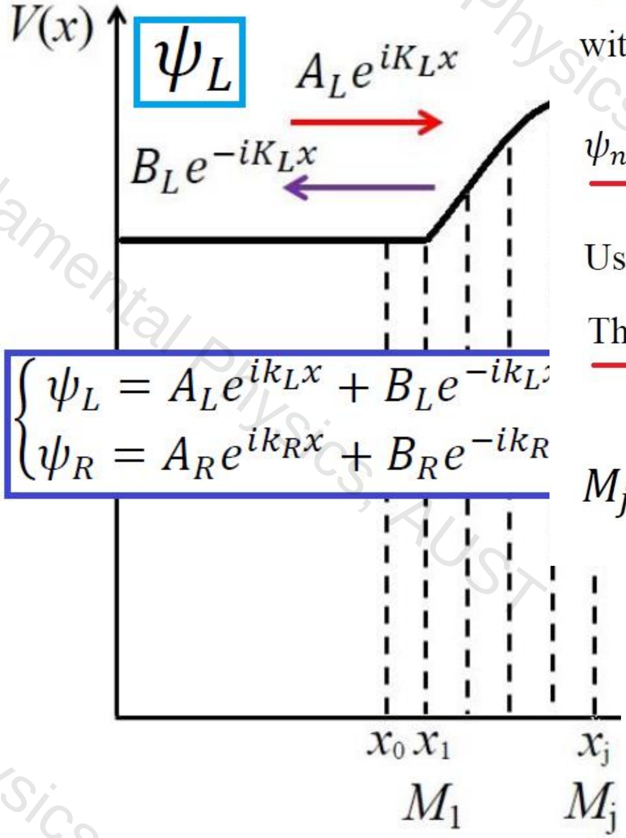
$$\kappa^2=k_0^2-k^2,\quad \delta=\frac{\kappa}{k}-\frac{k}{\kappa}$$

任意形状的双势垒，发生共振隧穿的物理条件？

发生共振隧穿的物理条件

PRB 36, 4203 (1987)

任意形状势垒的隧穿 — 转移矩阵方法



$$\begin{cases}\psi_L = A_L e^{ik_L x} + B_L e^{-ik_L} \\ \psi_R = A_R e^{ik_R x} + B_R e^{-ik_R}\end{cases}$$

Numerically, the diffusion path can be divided into (*N*+1) parts/regions with magnitude V_n, for the *n*th region:

$$\psi_n(x) \sim A_n e^{k_n x} + B_n e^{-k_n x}, \;\; k_n = \sqrt{2m(V_n - E)/\hbar^2},$$

Using the condition of wave function match ($\psi_n = \psi_{n+1}$; $\psi_n' = \psi_{n+1}'$) at x_{n+1}.

The *j*th transition matrix

$$M_j = \frac{1}{2k_j}\begin{pmatrix}(k_j + k_{j-1})e^{-(k_j - k_{j-1})x_j} & (k_j - k_{j-1})e^{-(k_j + k_{j-1})x_j} \\ (k_j - k_{j-1})e^{(k_j + k_{j-1})x_j} & (k_j + k_{j-1})e^{(k_j - k_{j-1})x_j}\end{pmatrix}$$

Then the chain product of M_j gives the transfer matrix M

$$M = M_N M_{N-1} \dots M_j \dots M_2 M_1 \equiv \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{pmatrix}$$

The transmission probability **at incident energy *E***:

$$T_r(E) = \left|\frac{A_R}{A_L}\right|^2 \times \frac{K_R}{K_L} = \frac{|M|^2}{|m_{22}|^2} \times \frac{K_R}{K_L}$$

$$\begin{pmatrix}A_R\end{pmatrix} \; M \begin{pmatrix}A_L\end{pmatrix} \; ($$

TMM: Numerically Accurate at Quantum Level!!

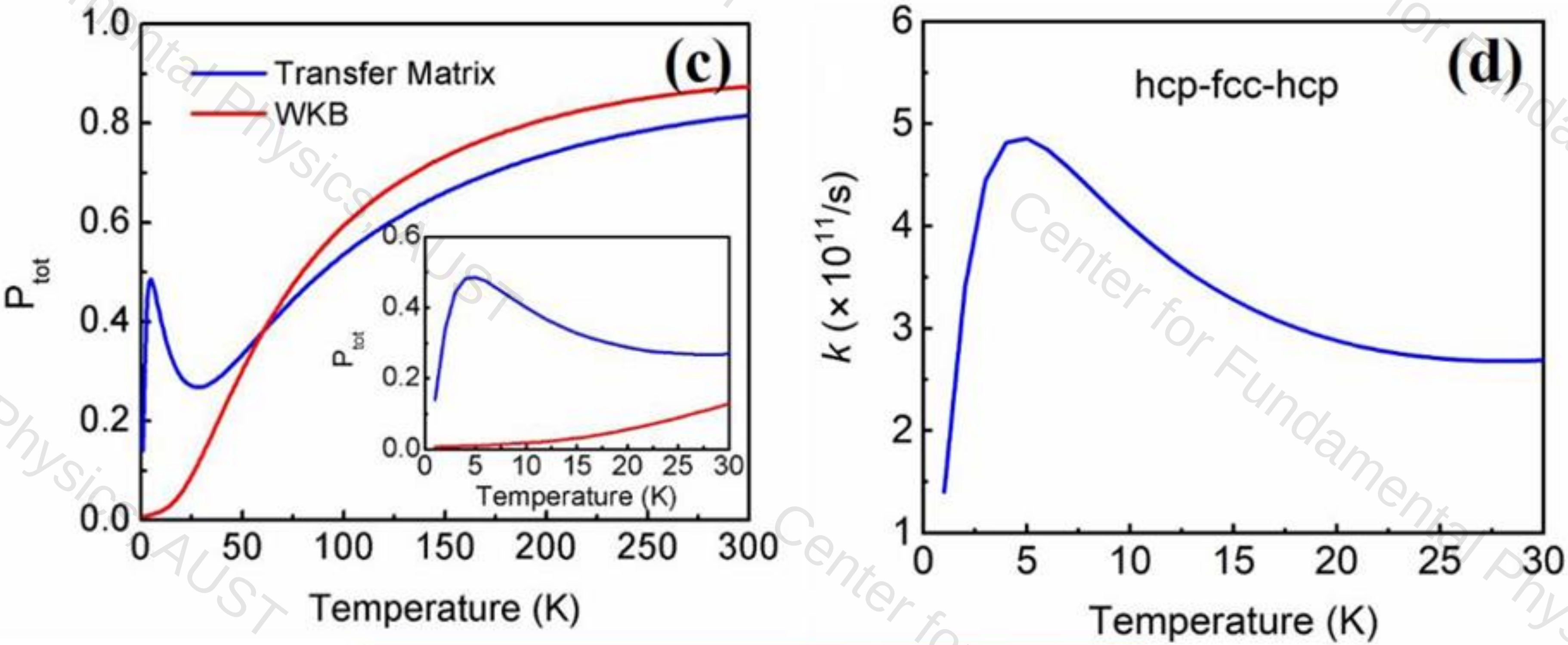
J. Phys. Chem. C 2021, 125, 464−480

$$T_r(E) = \frac{1}{|m_{22}|^2}$$

氢原子在Pt(111)面的共振隧穿

$$P_{tot}(T) = \int_0^\infty p(E,T)T_r(E)dE$$

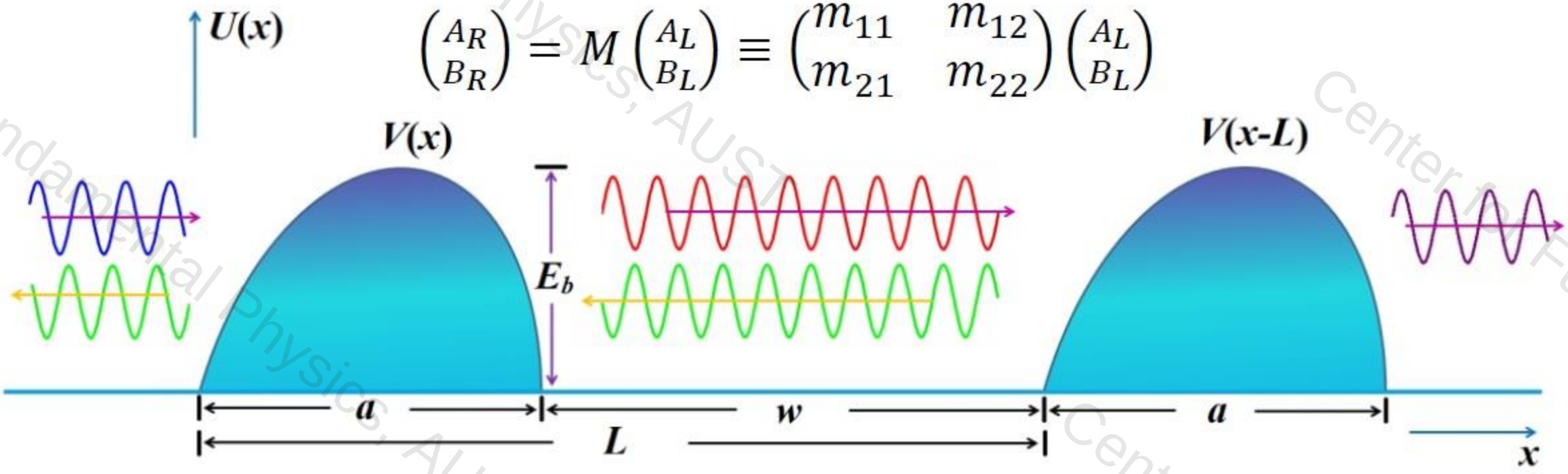
kinetic energy distribution $p(E,T) = 2\pi\left(\frac{1}{\pi k_B T}\right)^{3/2}\sqrt{E}e^{-E/k_B T}$



J. Phys. Chem. C 2021, 125, 464–480

H Diffusion along the path **hcp-fcc-hcp**: **Anomalous Rate of Diffusion!**

任意形状双势垒的共振隧穿



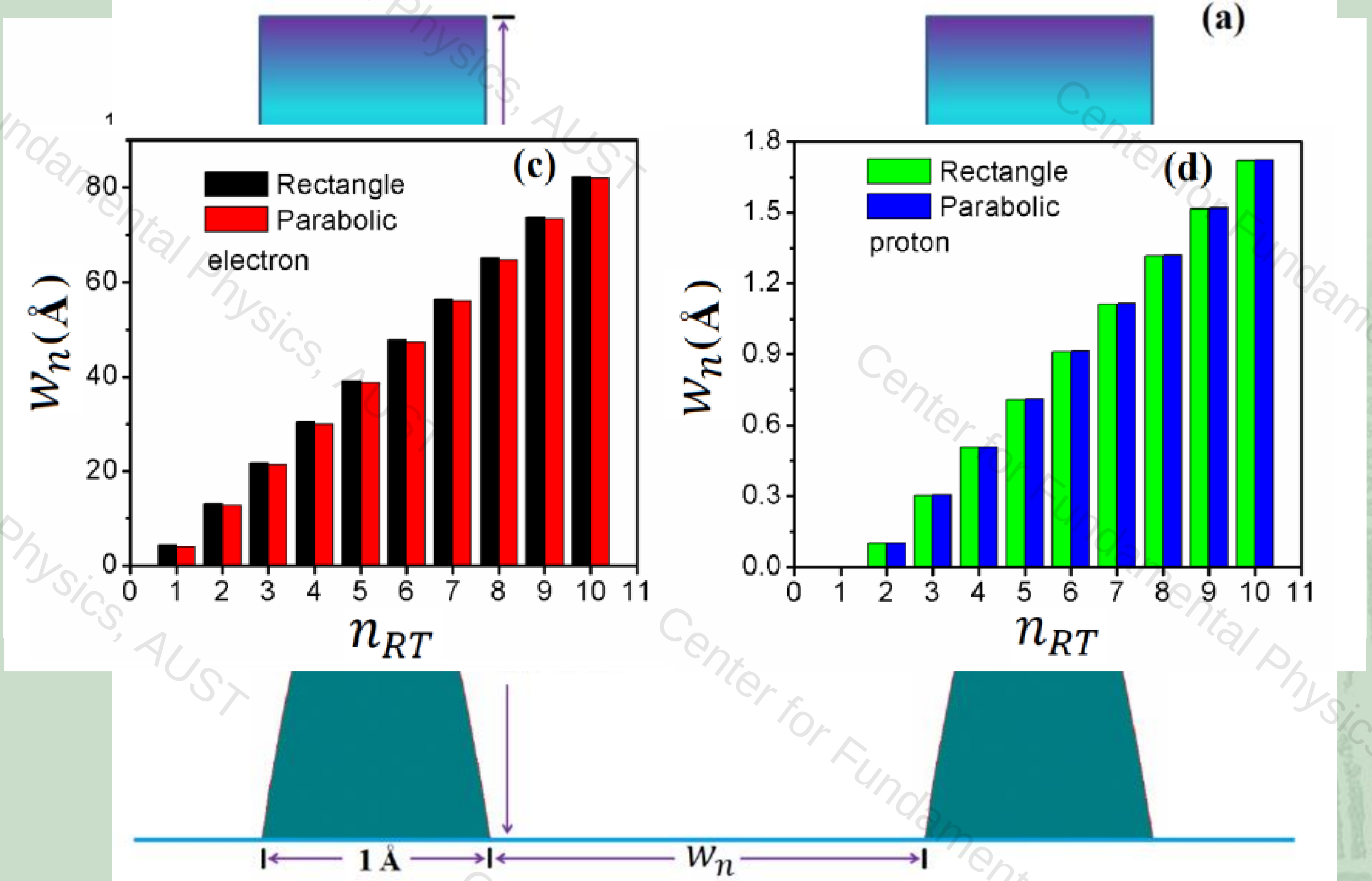
$$\begin{pmatrix} A_R \\ B_R \end{pmatrix} = M \begin{pmatrix} A_L \\ B_L \end{pmatrix} \equiv \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} A_L \\ B_L \end{pmatrix}$$

Theorem. – For any $E < E_b$, the transmission coefficient (tunneling probability) across a homo-structured double-barrier $T_{DB}(E;w) = 1$ at $w = \boxed{w_n = \frac{n\pi}{k} - \frac{\pi+\theta+2ka}{2k}}$, where $\theta = \arg({m_{11}}^2)$, n (referred to as resonance number) belongs to integers.

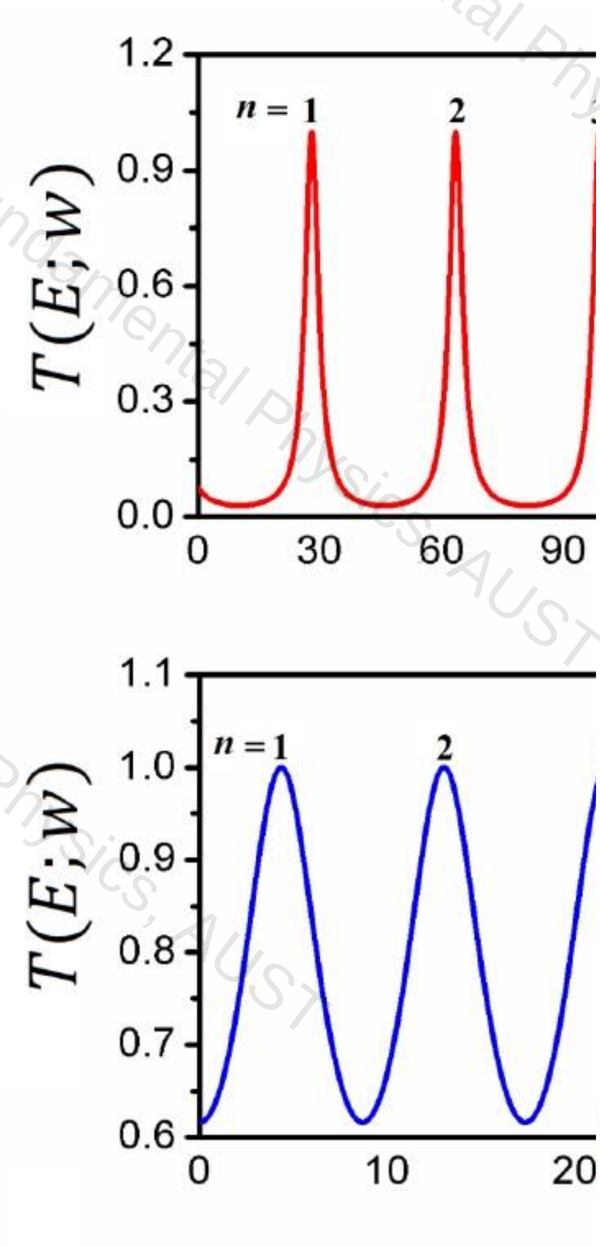
对任意形状的同构双势垒，只要适当调整势垒间距，就可以实现共振隧穿！

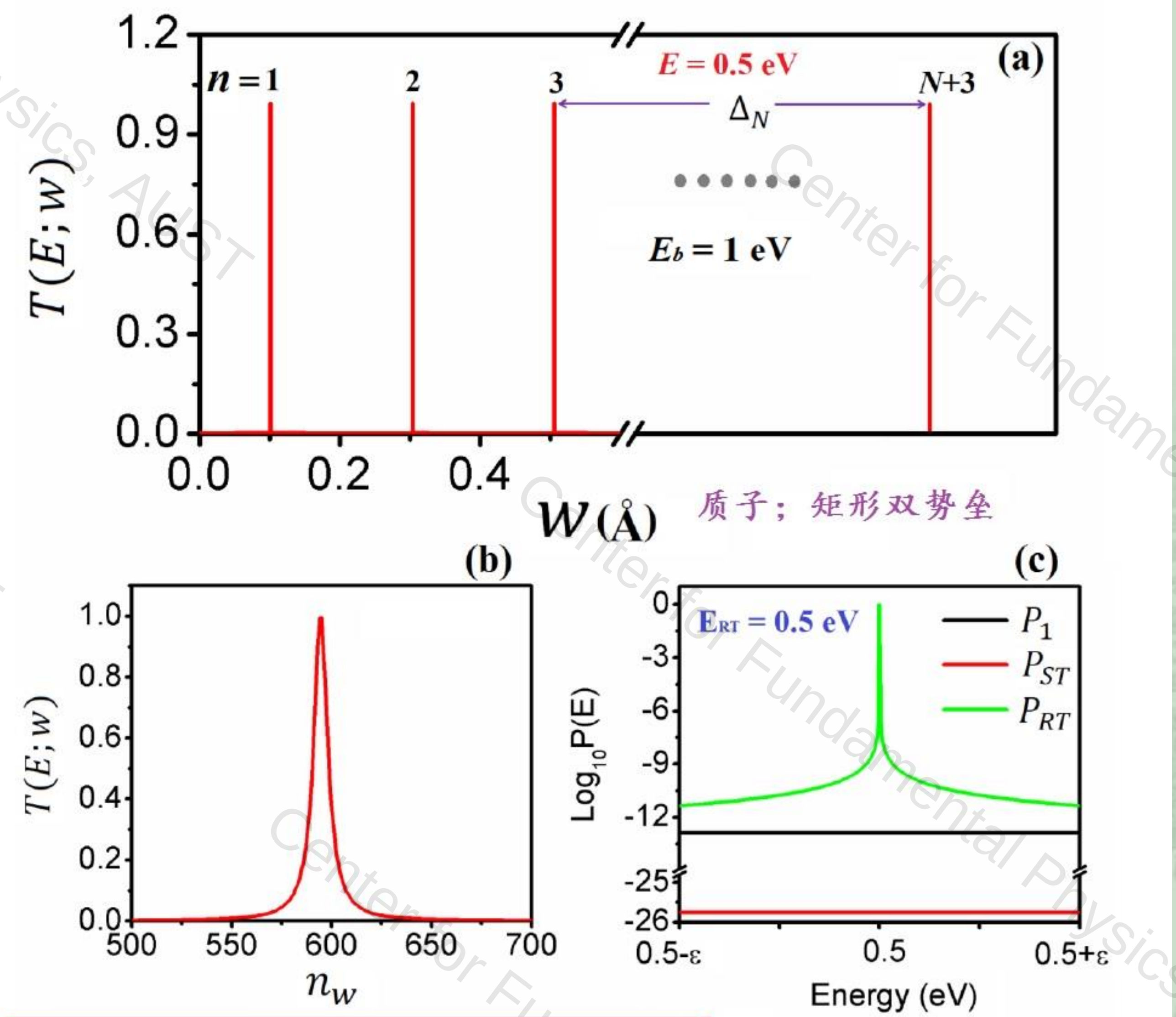
Phys. Rev. Res. **6**, 013087(2024)

双势垒共振隧穿的一些特征



共振峰位的特征分析





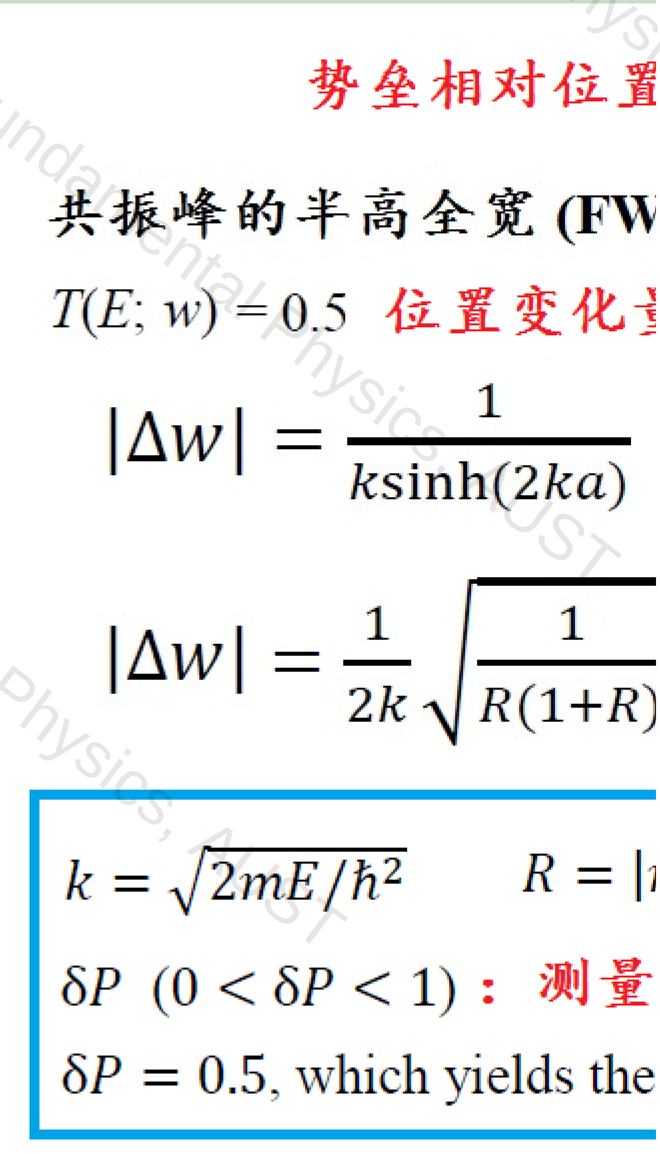
质子；矩形双势垒

$w = (n_w - n_p)\times\Delta l + w_n$, $n_p = 596$ and $\Delta l = 10^{-15}$ Å

$w = 20.137016632763302$ Å

$\varepsilon = 10^{-10}$ eV

位置和能量展宽的影响



势垒相对位置

共振峰的半高全宽 (FW

$T(E;\, w) = 0.5$ 位置变化

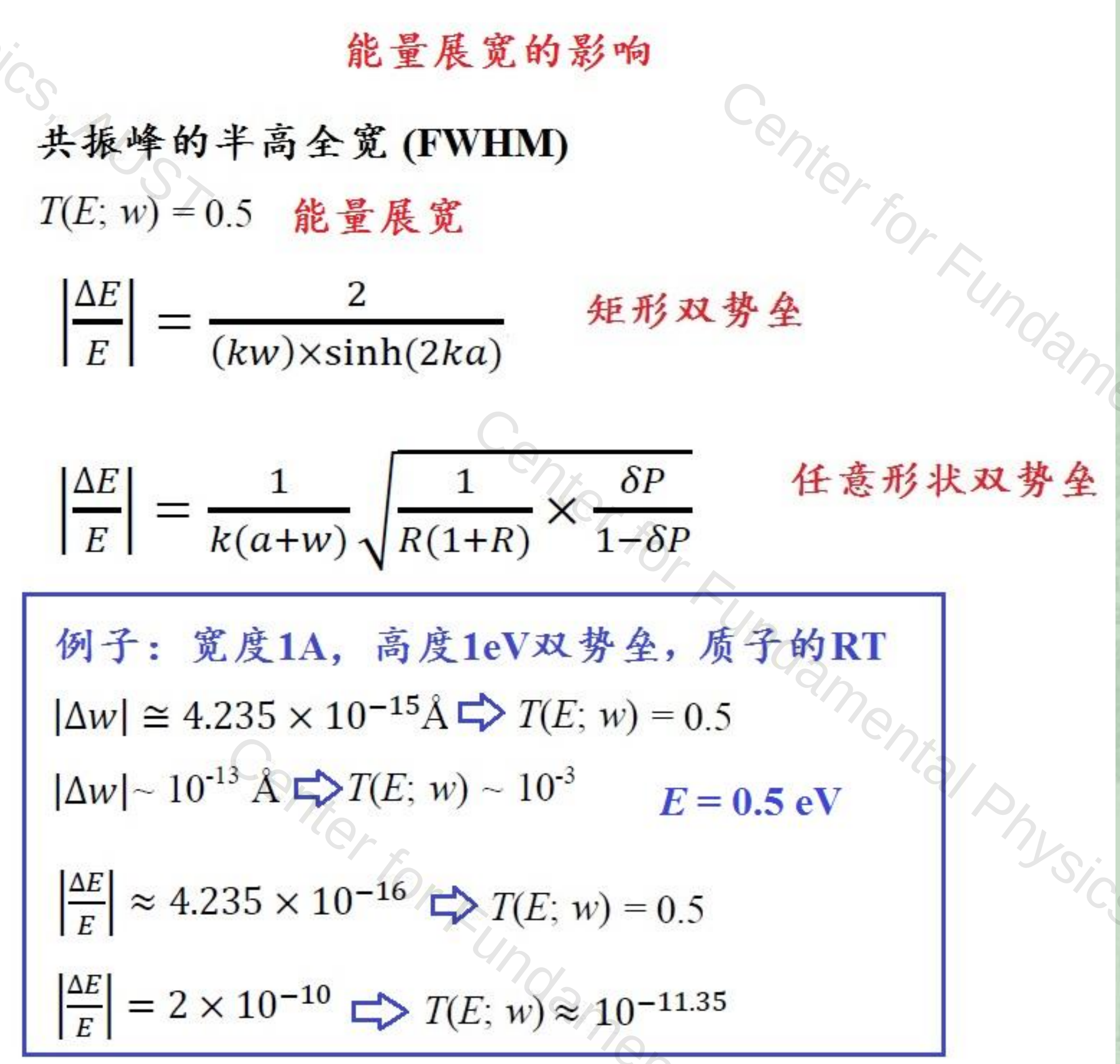
$$|\Delta w| = \frac{1}{k\sinh(2ka)}$$

$$|\Delta w| = \frac{1}{2k}\sqrt{\frac{1}{R(1+R)}}$$

$k = \sqrt{2mE/\hbar^2}$　　$R = |$

δP $(0 < \delta P < 1)$： 测量

$\delta P = 0.5$, which yields the



能量展宽的影响

共振峰的半高全宽 (FWHM)

$T(E;\, w) = 0.5$ 能量展宽

$$\left|\frac{\Delta E}{E}\right| = \frac{2}{(kw)\times\sinh(2ka)}$$ 矩形双势垒

$$\left|\frac{\Delta E}{E}\right| = \frac{1}{k(a+w)}\sqrt{\frac{1}{R(1+R)}\times\frac{\delta P}{1-\delta P}}$$ 任意形状双势垒

例子：宽度1A，高度1eV双势垒，质子的RT

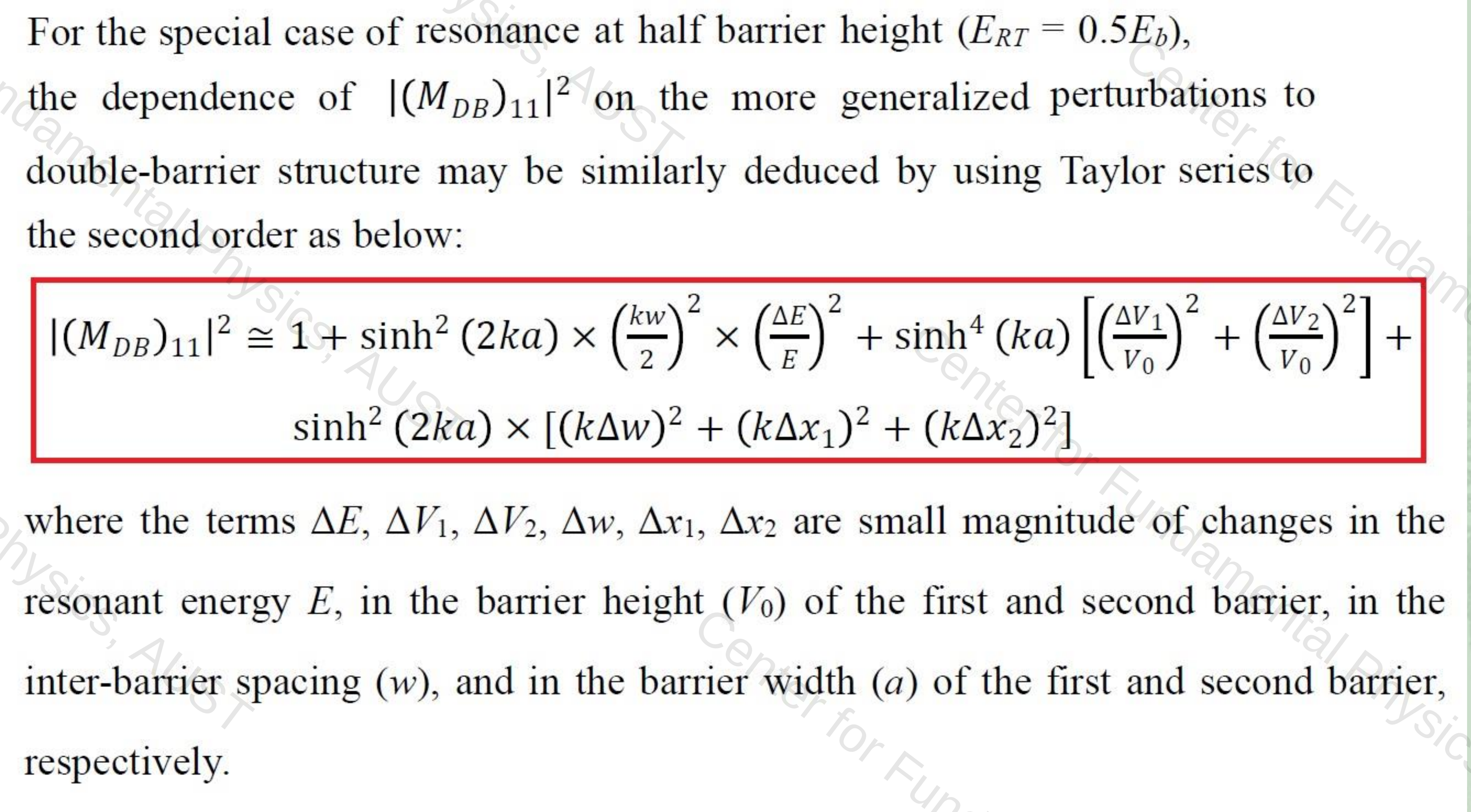
$|\Delta w| \cong 4.235\times 10^{-15}$Å ⇨ $T(E;\, w) = 0.5$

$|\Delta w| \sim 10^{-13}$ Å ⇨ $T(E;\, w) \sim 10^{-3}$　　**$E = 0.5$ eV**

$\left|\frac{\Delta E}{E}\right| \approx 4.235\times 10^{-16}$ ⇨ $T(E;\, w) = 0.5$

$\left|\frac{\Delta E}{E}\right| = 2\times 10^{-10}$ ⇨ $T(E;\, w) \approx 10^{-11.35}$

描写微扰的一般表达式

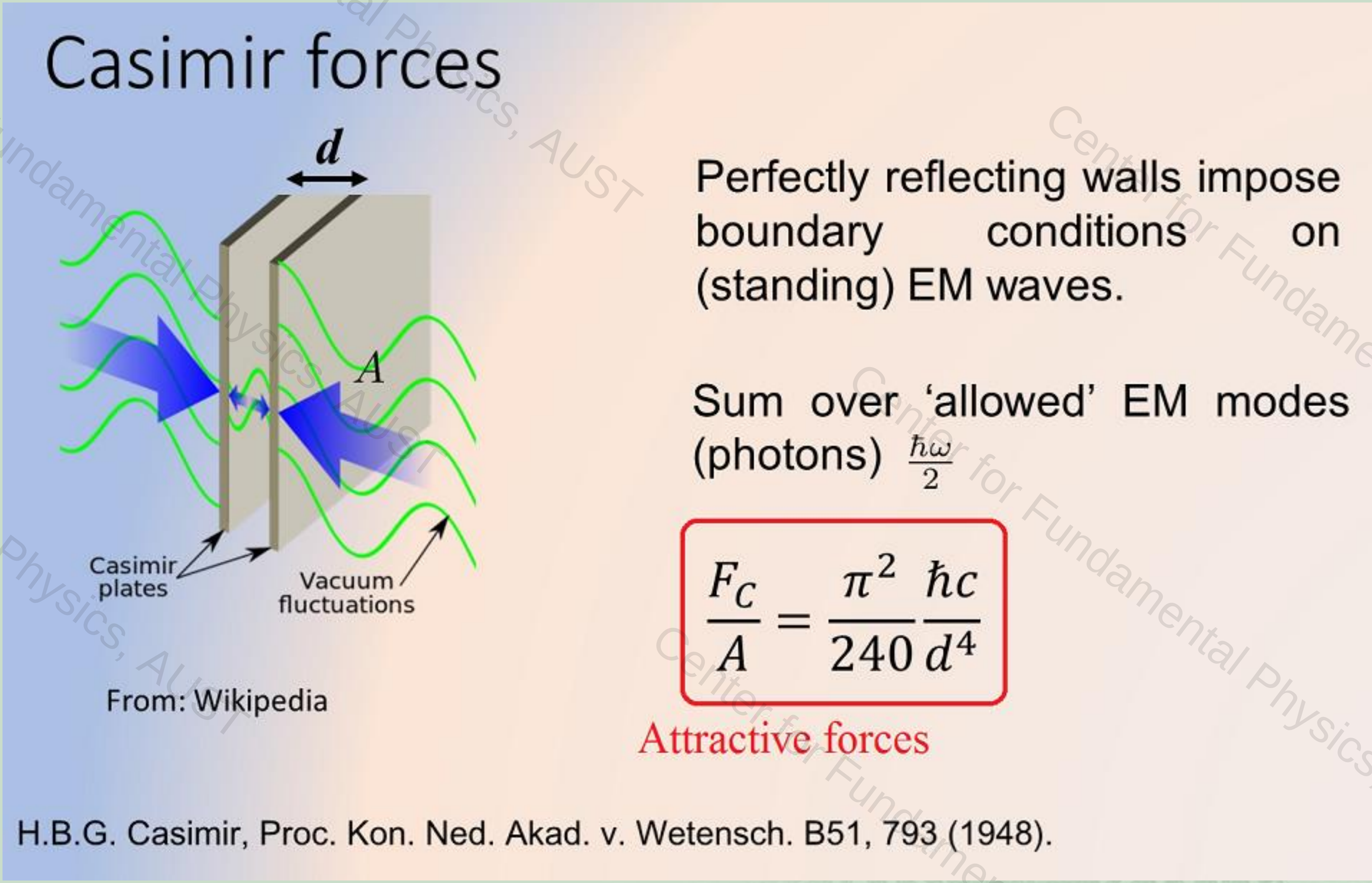


For the special case of resonance at half barrier height ($E_{RT} = 0.5E_b$), the dependence of $|(M_{DB})_{11}|^2$ on the more generalized perturbations to double-barrier structure may be similarly deduced by using Taylor series to the second order as below:

$$\boxed{\begin{aligned}|(M_{DB})_{11}|^2 \cong 1 + \sinh^2(2ka) \times \left(\frac{kw}{2}\right)^2 \times \left(\frac{\Delta E}{E}\right)^2 + \sinh^4(ka)\left[\left(\frac{\Delta V_1}{V_0}\right)^2 + \left(\frac{\Delta V_2}{V_0}\right)^2\right] + \\ \sinh^2(2ka) \times [(k\Delta w)^2 + (k\Delta x_1)^2 + (k\Delta x_2)^2]\end{aligned}}$$

where the terms ΔE, ΔV_1, ΔV_2, Δw, Δx_1, Δx_2 are small magnitude of changes in the resonant energy E, in the barrier height (V_0) of the first and second barrier, in the inter-barrier spacing (w), and in the barrier width (a) of the first and second barrier, respectively.

Casimir力的影响的评估



Casimir forces

From: Wikipedia

Perfectly reflecting walls impose boundary conditions on (standing) EM waves.

Sum over 'allowed' EM modes (photons) $\frac{\hbar\omega}{2}$

$$\boxed{\frac{F_C}{A} = \frac{\pi^2}{240}\frac{\hbar c}{d^4}}$$

Attractive forces

H.B.G. Casimir, Proc. Kon. Ned. Akad. v. Wetensch. B51, 793 (1948).

Casimir forces typically operate in the range of submicron to micron

The energy change per unit surface area may be calculated as follows:

$$\boxed{\Delta E_0(d) = -\frac{\pi^2}{720} \times \frac{\hbar c}{d^3},}$$

For a double-barrier with cross-section area A, the potential change at a distance of x due to Casimir effect is

$$\Delta V_0(x) = -A \times \frac{\pi^2}{720} \times \frac{\hbar c}{x^3}, \text{ where } w_n \le x \le w_n + a + b, \;\; w_n \gg (a+b)$$

⇨ $\Delta V_0(w_n) = -A \times \frac{\pi^2}{720} \times \frac{\hbar c}{{w_n}^3}.$ Given that $A \sim {w_n}^2$, then

$$\boxed{\Delta V_0(w_n) = -\frac{\pi^2}{720} \times \frac{\hbar c}{w_n}}$$

It follows that $\Delta V_0 \cong 2.7$ meV and 27 meV, respectively, for $w_n = 1\,\mu$m and $0.1\,\mu$m, respectively.

$V_0 = 1$ eV, 影响可以忽略

引力波的影响的评估

假定引力波经过引起势垒区域均匀的形变（形变量为h），对同构矩形双势垒

$$\Delta w = h \times w,\ \Delta x_1 = \Delta x_2 = h \times a;\quad h = 1 \times 10^{-21}$$

w and a are respectively the inter-barrier spacing and barrier width. In the case of $E_{RT} = 0.5E_b$,

$$\boxed{|(M_{DB})_{11}|^2 = 1 + \sinh^2(2ka) \times h^2 \times [(kw_n)^2 + 2(ka)^2]}$$

The significant change of $T(E)$, $T(E) = 0.5$, i.e., $\delta P = 0.5$ yields $\underline{|(M_{DB})_{11}|^2 = 2}$, which gives that

$$\boxed{\sinh^2(2ka) \times h^2 \times [(kw_n)^2 + 2(ka)^2] = 1} \qquad (12)$$

Let $\underline{\chi = 2ka}$, recalling $\underline{2kw_n = (2n-1)\pi}$ for resonance at $E_{RT} = 0.5E_b$, Eq. (12) may be rewritten as

$$\boxed{\sinh(\chi)\sqrt{[(2n-1)\pi]^2 + 2\chi^2} = 2h^{-1},} \qquad (13)$$

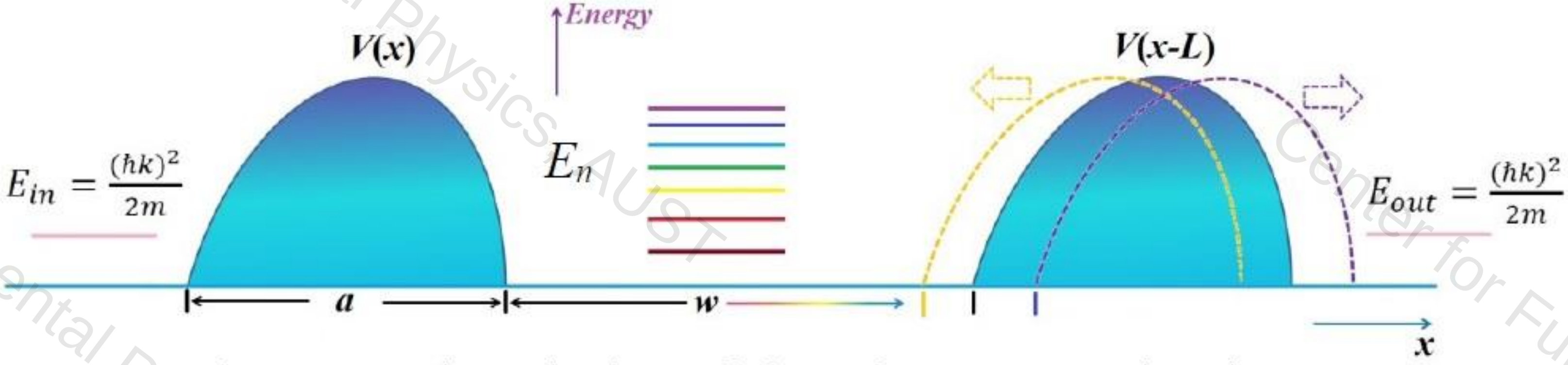
where $k = \frac{\sqrt{2mE}}{\hbar} = \frac{\sqrt{mE_b}}{\hbar}$. The nth root $\chi_n = 2ka$, puts Strong constraint on the barrier height (E_b) and barrier width (a) at inter-barrier spacing w_n. For instance, $\chi_1 \cong 45.5375$ at $w_1 \cong 4.33607172$ Å and $E_b = 1$ eV require that $a \cong 62.85151757$ Å for a significant probability drop ($\delta P = 0.5$) of electron tunneling

Table I. Energetic and geometric parameters for the resonant tunneling of electrons across homo-structured rectangular double-barriers, and the decrease of transmission probability (δP) due to the strain ($h = 1\times10^{-21}$) induced by gravitational waves. For all the double-barriers, resonance takes place at $E_{\mathrm{RT}} = 0.5E_{\mathrm{b}}$. The number of digits are set by $|\Delta w|$, $|\Delta E|$ when $T(E_{\mathrm{RT}}) = 0.5$.

显著的引力波效应，需要有极为精确设计的双势垒结构！

n	χ_n	E_b (eV)	w_{n} (Å)	a (Å)	δP
1	45.5375	1	4.336	10	1.091×10^{-32}
		1	4.3360717180254546576	62.8515175683137030659	0.5
		39.5031326064004559839	0.6898913321086561634	10	0.5
10	45.2686	1	82.385	10	1.872×10^{-31}
		1	82.3853626424836420483	62.4803778905949016575	0.5
		39.0379762135153995928	13.1079353100644677709	10	0.5
100	43.2978	1	862.878	10	1.941×10^{-29}
		1	862.8782718870654662168	59.7602511637514837162	0.5
		35.7128761915465986476	144.3900008925093061407	10	0.5

势阱能级展宽的影响



Γ_n: the energy broadening of the nth resonance level (E_n)

Near Resonance, Breit-Wigner formula $T(E) \cong \frac{\Gamma_n^2}{(E-E_n)^2+\Gamma_n^2}$

When $\Gamma_n > \Delta E_{\text{intr}}$, intrinsic FWHM. For resonance at $2E = E_b$,

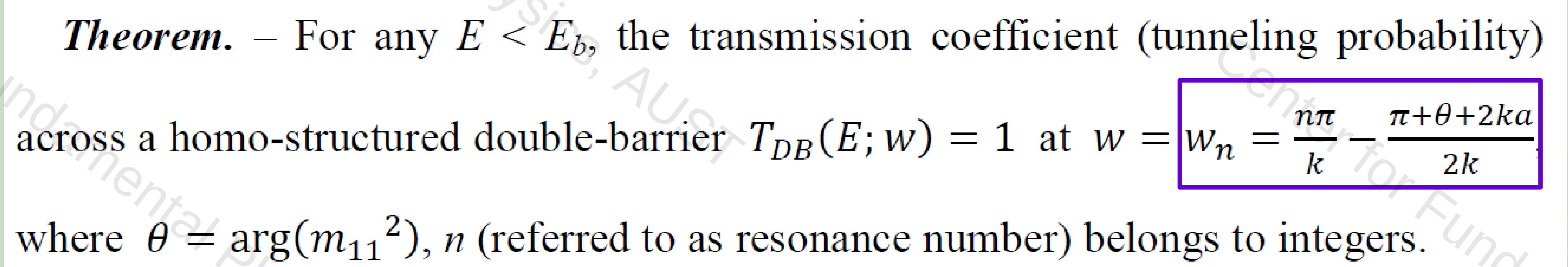
$$|(M_{DB})_{11}|^2 \cong 1 + \sinh^2\left(2k(1-\frac{\Gamma_n}{E_b})a\right) \times \left(k(1-\frac{\Gamma_n}{E_b})\Delta w\right)^2$$

Consequently, the deviation Δw which defines the FWHM at length scale is

$$\boxed{|\Delta w| = \frac{1}{k(1-\frac{\Gamma_n}{E_b})\sinh(2k(1-\frac{\Gamma_n}{E_b})a)}.}$$

势阱内准束缚态能级的展宽，会使得发生有效共振隧穿的几何约束放松

共振隧穿和实空间连续性



Theorem. – For any $E < E_b$, the transmission coefficient (tunneling probability) across a homo-structured double-barrier $T_{DB}(E;w) = 1$ at $w = \boxed{w_n = \frac{n\pi}{k} - \frac{\pi+\theta+2ka}{2k}}$, where $\theta = \arg({m_{11}}^2)$, n (referred to as resonance number) belongs to integers.

- 定理成立的内含前提条件 — 实空间是连续的！

换言之，我们总可以通过有限的实际操作，以任意小的变化步长，得到理论预测的势垒间距w_n，从而实现共振隧穿

对于宏观世界，实空间的连续性是不证自明的

对于微观世界，实空间的连续性尚存疑问；一些试图统一量子力学和引力的理论认为，存在一个最小空间尺度，通常取为普朗克长度：

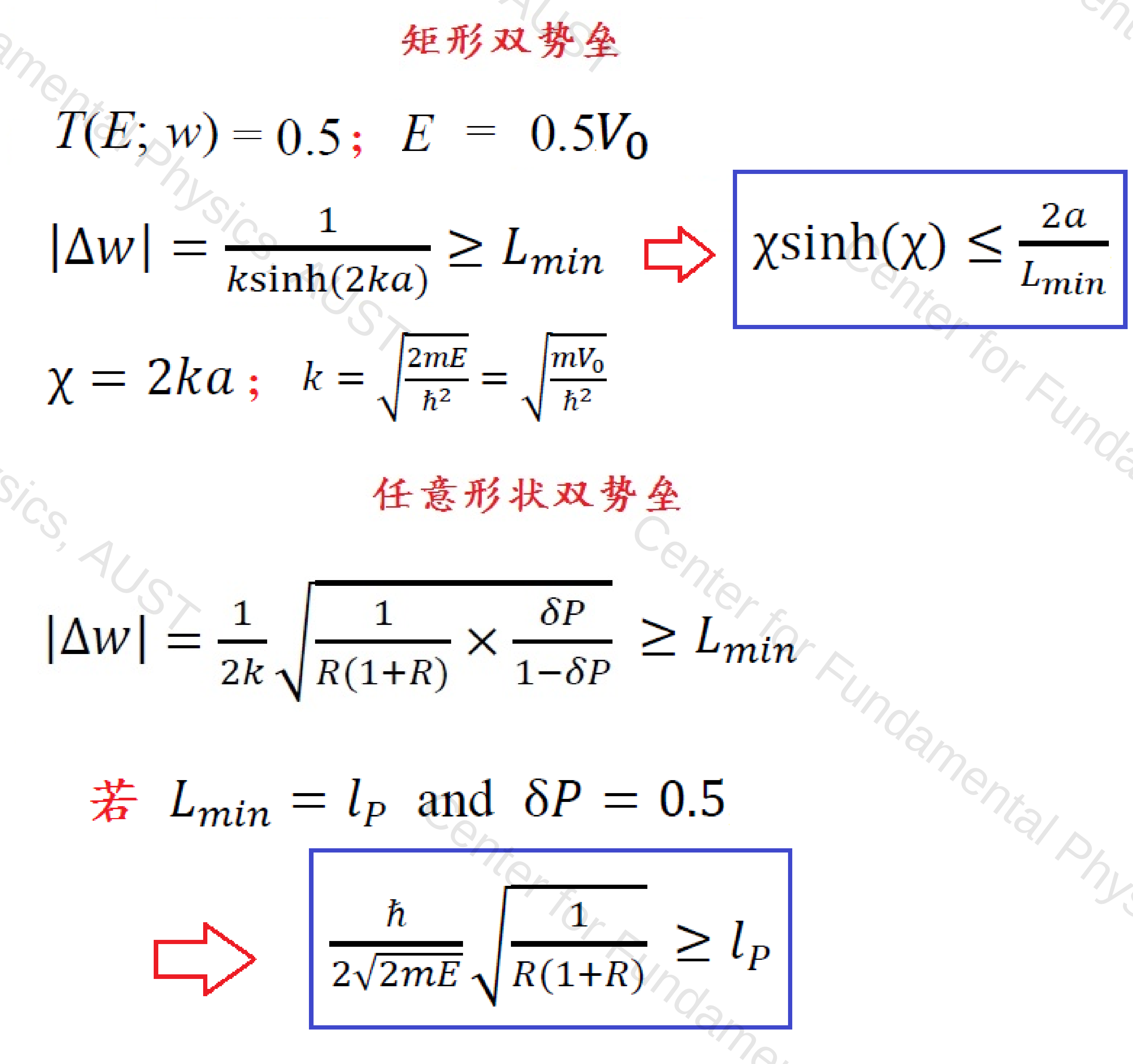
$$\boxed{l_P = \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-35}\ m}$$

- 最小空

换言之，

能以最小

距w。这



矩形双势垒

$$T(E;w)=0.5\text{；}\quad E=0.5V_0$$

$$|\Delta w|=\frac{1}{k\sinh(2ka)}\geq L_{min}\Rightarrow\boxed{\chi\sinh(\chi)\leq\frac{2a}{L_{min}}}$$

$$\chi=2ka\text{；}\quad k=\sqrt{\frac{2mE}{\hbar^2}}=\sqrt{\frac{mV_0}{\hbar^2}}$$

任意形状双势垒

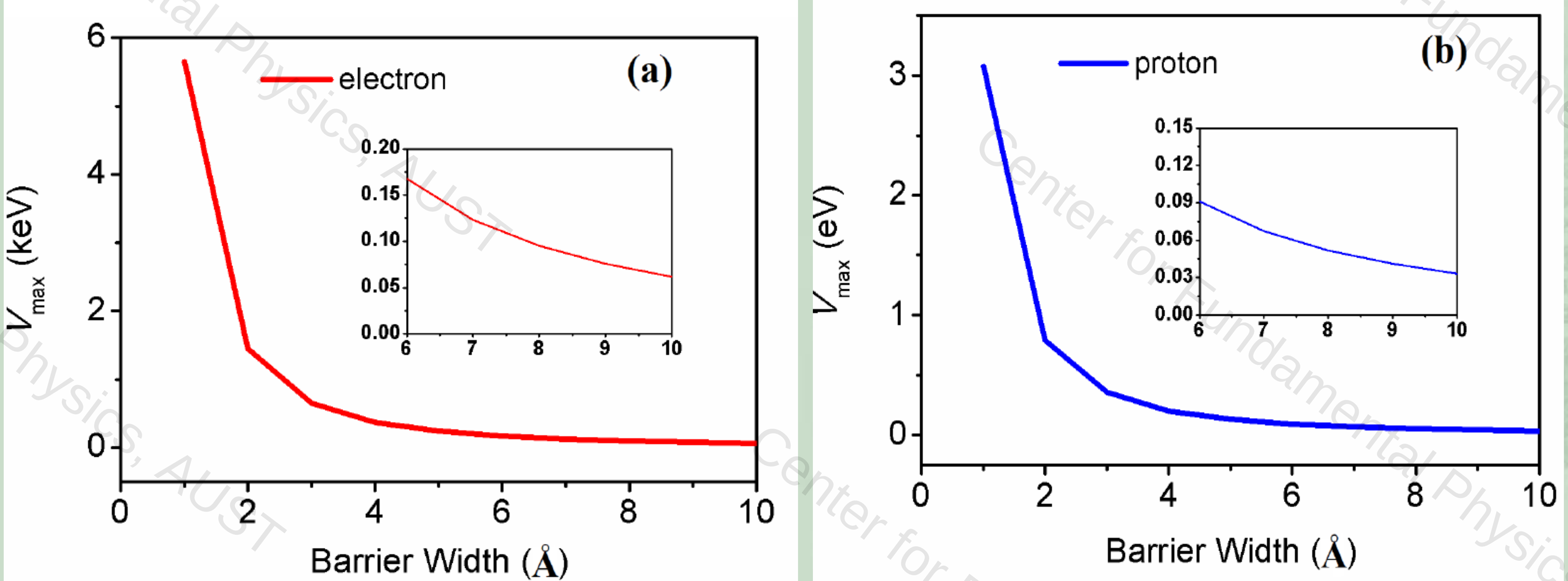
$$|\Delta w|=\frac{1}{2k}\sqrt{\frac{1}{R(1+R)}\times\frac{\delta P}{1-\delta P}}\geq L_{min}$$

若 $L_{min}=l_P$ and $\delta P=0.5$

$$\Rightarrow\boxed{\frac{\hbar}{2\sqrt{2mE}}\sqrt{\frac{1}{R(1+R)}}\geq l_P}$$

共振隧穿的势垒高度上限

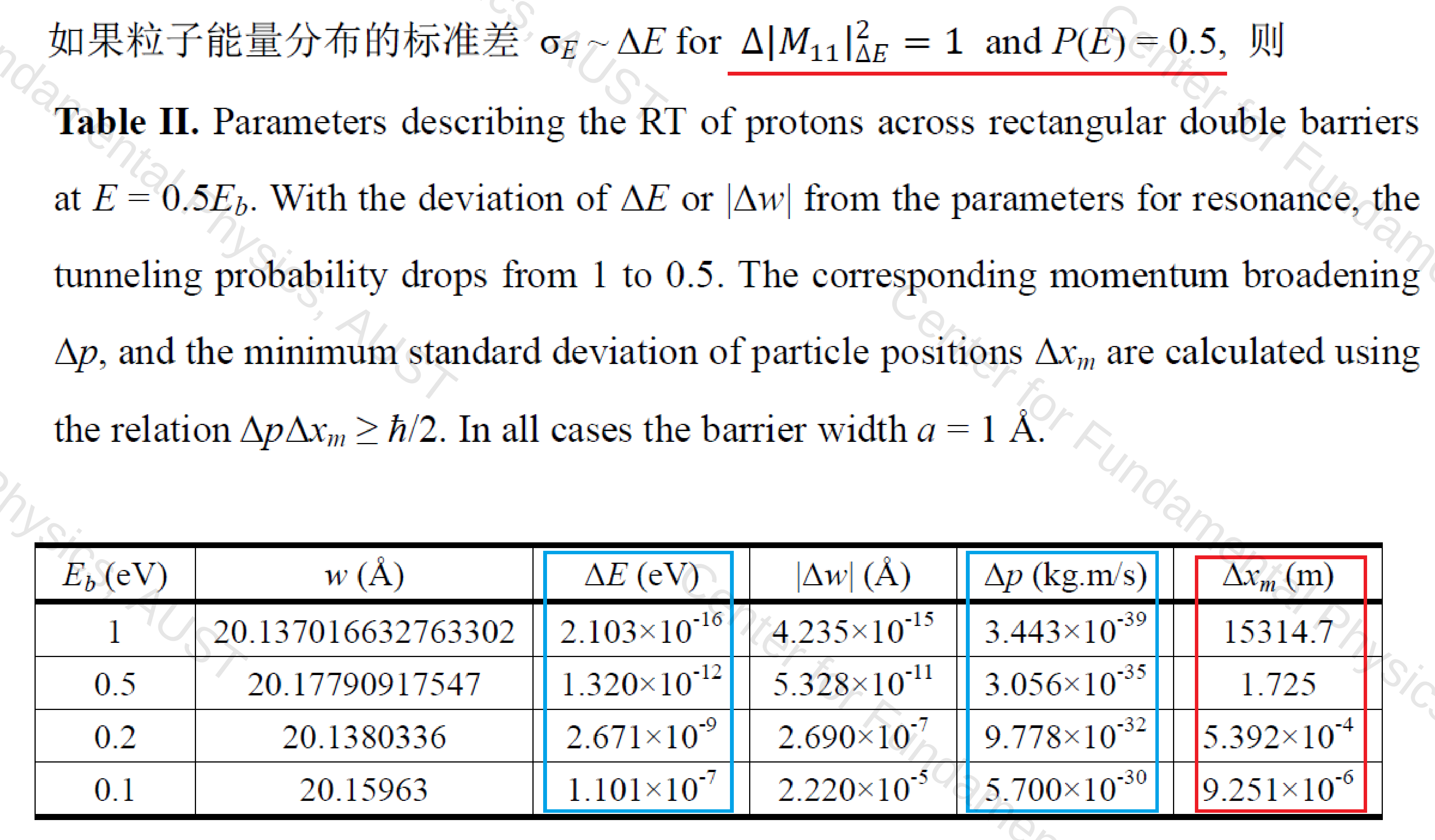
- 矩形双势垒，$E = 0.5V_0$ (半势垒高度), $L_{min} = l_P$



$V_0 > V_{max}$, 无可观测共振隧穿！

不确定原理的物理约束

- 不确定原理 (uncertainty principle) 的约束



如果粒子能量分布的标准差 $\sigma_E \sim \Delta E$ for $\Delta|M_{11}|^2_{\Delta E} = 1$ and $P(E) = 0.5$, 则

Table II. Parameters describing the RT of protons across rectangular double barriers at $E = 0.5E_b$. With the deviation of ΔE or $|\Delta w|$ from the parameters for resonance, the tunneling probability drops from 1 to 0.5. The corresponding momentum broadening Δp, and the minimum standard deviation of particle positions Δx_m are calculated using the relation $\Delta p \Delta x_m \geq \hbar/2$. In all cases the barrier width a = 1 Å.

E_b (eV)	w (Å)	ΔE (eV)	$\|\Delta w\|$ (Å)	Δp (kg.m/s)	Δx_m (m)
1	20.137016632763302	2.103×10^{-16}	4.235×10^{-15}	3.443×10^{-39}	15314.7
0.5	20.17790917547	1.320×10^{-12}	5.328×10^{-11}	3.056×10^{-35}	1.725
0.2	20.1380336	2.671×10^{-9}	2.690×10^{-7}	9.778×10^{-32}	5.392×10^{-4}
0.1	20.15963	1.101×10^{-7}	2.220×10^{-5}	5.700×10^{-30}	9.251×10^{-6}

不确定原理的物理约束—解决方案

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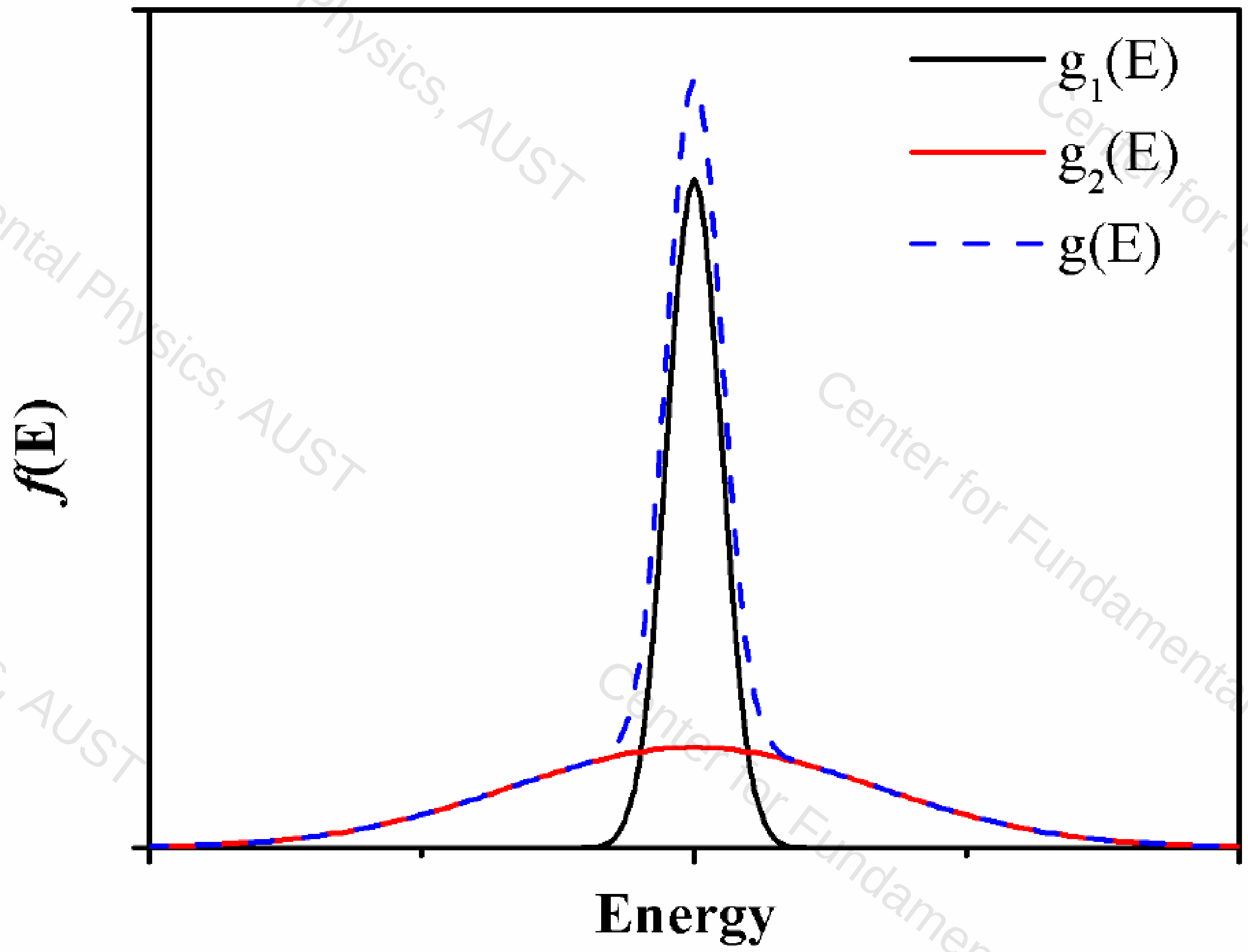


$\frac{\hbar}{2}$

=

实验检验体系的物理约束

- 很小的ΔE: 入射粒子束具有高度能量单色性
- 量子干涉: 入射粒子束波函数的高度相干性

满足条件的物理体系 (动量空间的凝聚):
1、超导电子对
2、玻色爱因斯坦凝聚体

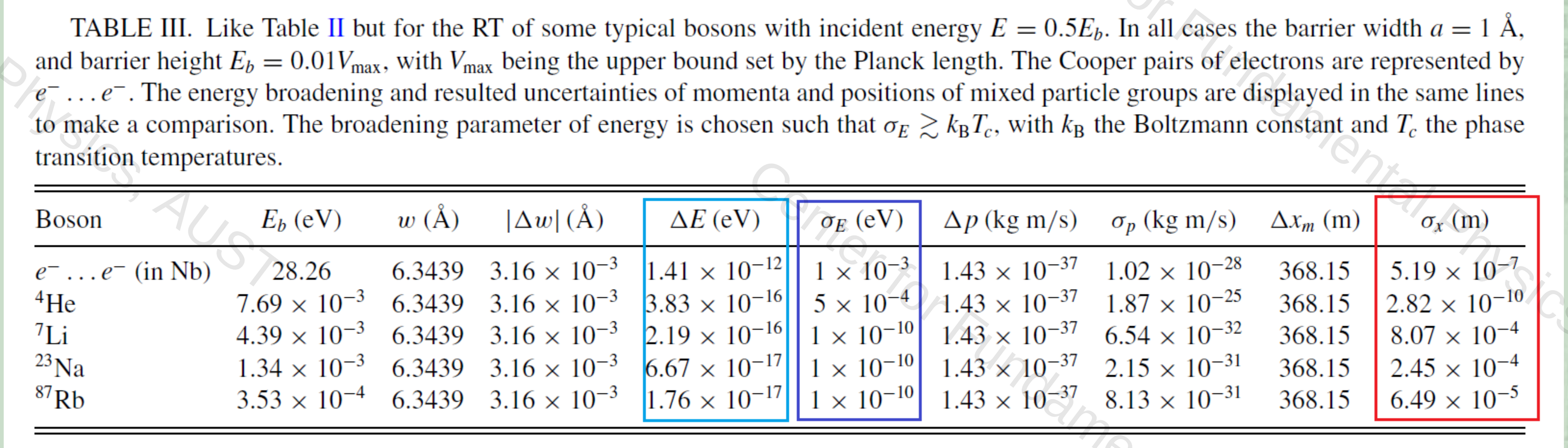
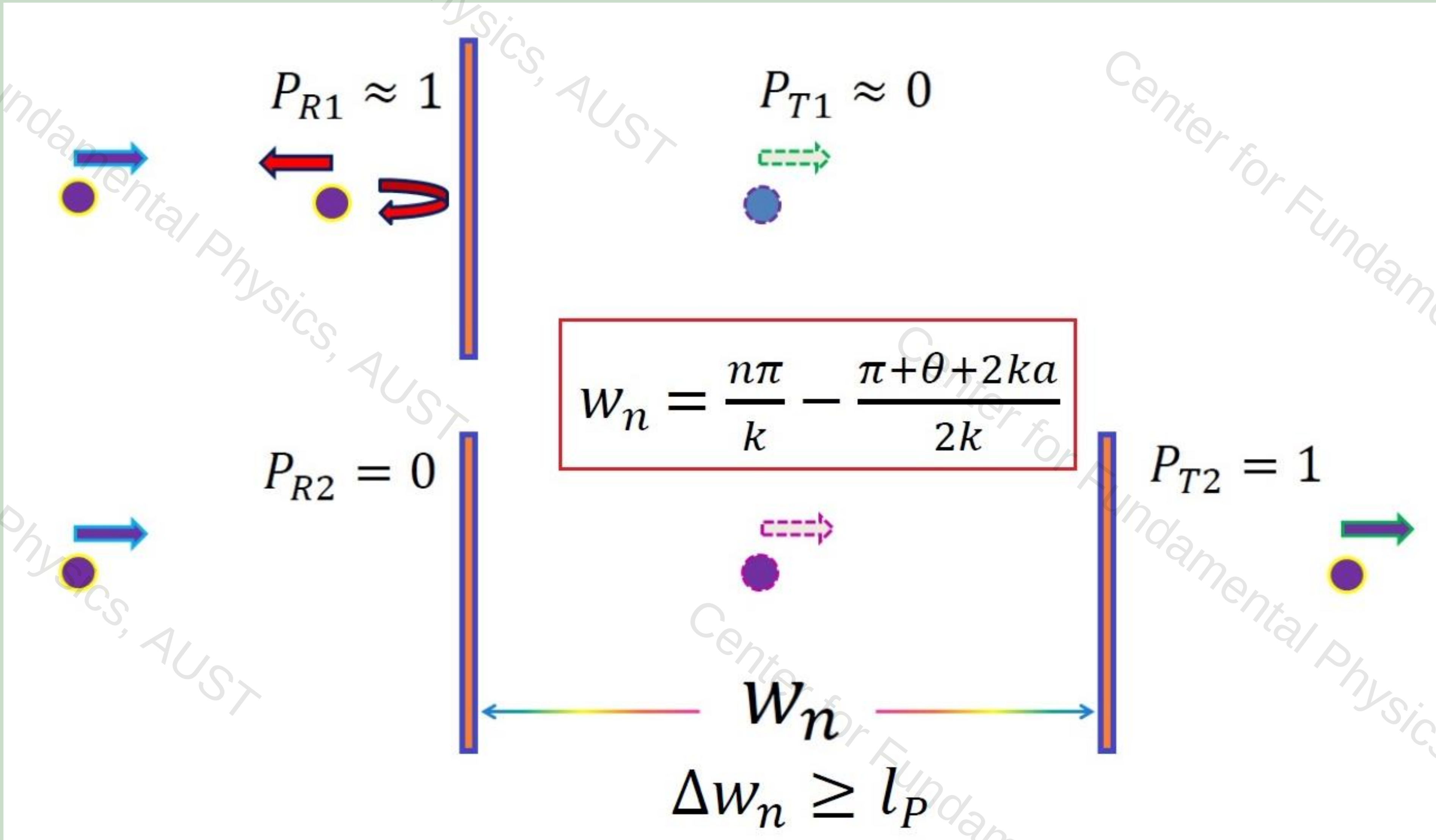


TABLE III. Like Table II but for the RT of some typical bosons with incident energy $E = 0.5E_b$. In all cases the barrier width $a = 1$ Å, and barrier height $E_b = 0.01V_{\max}$, with $V_{\max}$ being the upper bound set by the Planck length. The Cooper pairs of electrons are represented by $e^- \ldots e^-$. The energy broadening and resulted uncertainties of momenta and positions of mixed particle groups are displayed in the same lines to make a comparison. The broadening parameter of energy is chosen such that $\sigma_E \gtrsim k_B T_c$, with k_B the Boltzmann constant and T_c the phase transition temperatures.

Boson	E_b (eV)	w (Å)	$\lvert\Delta w\rvert$ (Å)	ΔE (eV)	σ_E (eV)	Δp (kg m/s)	σ_p (kg m/s)	Δx_m (m)	σ_x (m)
$e^- \ldots e^-$ (in Nb)	28.26	6.3439	3.16×10^{-3}	1.41×10^{-12}	1×10^{-3}	1.43×10^{-37}	1.02×10^{-28}	368.15	5.19×10^{-7}
^{4}He	7.69×10^{-3}	6.3439	3.16×10^{-3}	3.83×10^{-16}	5×10^{-4}	1.43×10^{-37}	1.87×10^{-25}	368.15	2.82×10^{-10}
^{7}Li	4.39×10^{-3}	6.3439	3.16×10^{-3}	2.19×10^{-16}	1×10^{-10}	1.43×10^{-37}	6.54×10^{-32}	368.15	8.07×10^{-4}
^{23}Na	1.34×10^{-3}	6.3439	3.16×10^{-3}	6.67×10^{-17}	1×10^{-10}	1.43×10^{-37}	2.15×10^{-31}	368.15	2.45×10^{-4}
^{87}Rb	3.53×10^{-4}	6.3439	3.16×10^{-3}	1.76×10^{-17}	1×10^{-10}	1.43×10^{-37}	8.13×10^{-31}	368.15	6.49×10^{-5}

原子世界的崂山道士



$P_{R1} \approx 1$

$P_{T1} \approx 0$

$$w_n = \frac{n\pi}{k} - \frac{\pi+\theta+2ka}{2k}$$

$P_{R2} = 0$

$P_{T2} = 1$

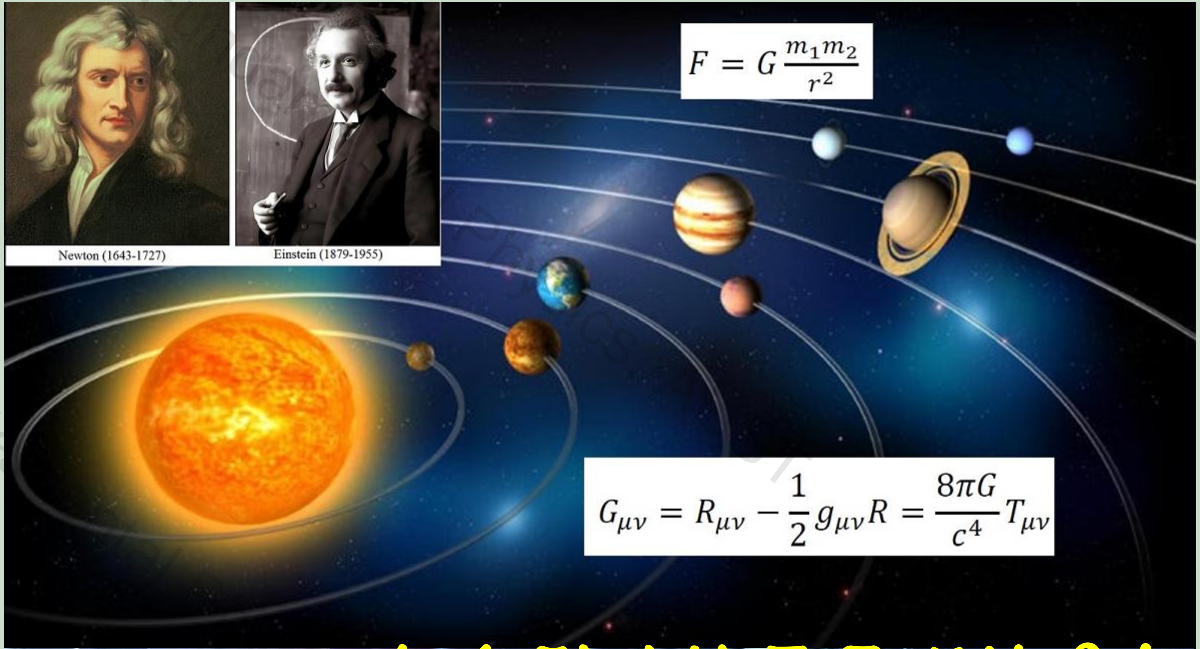
w_n

$\Delta w_n \geq l_P$

二十世纪物理学的三个主旋律

"Thematic Melodies of Twentieth Century Theoretical Physics:"

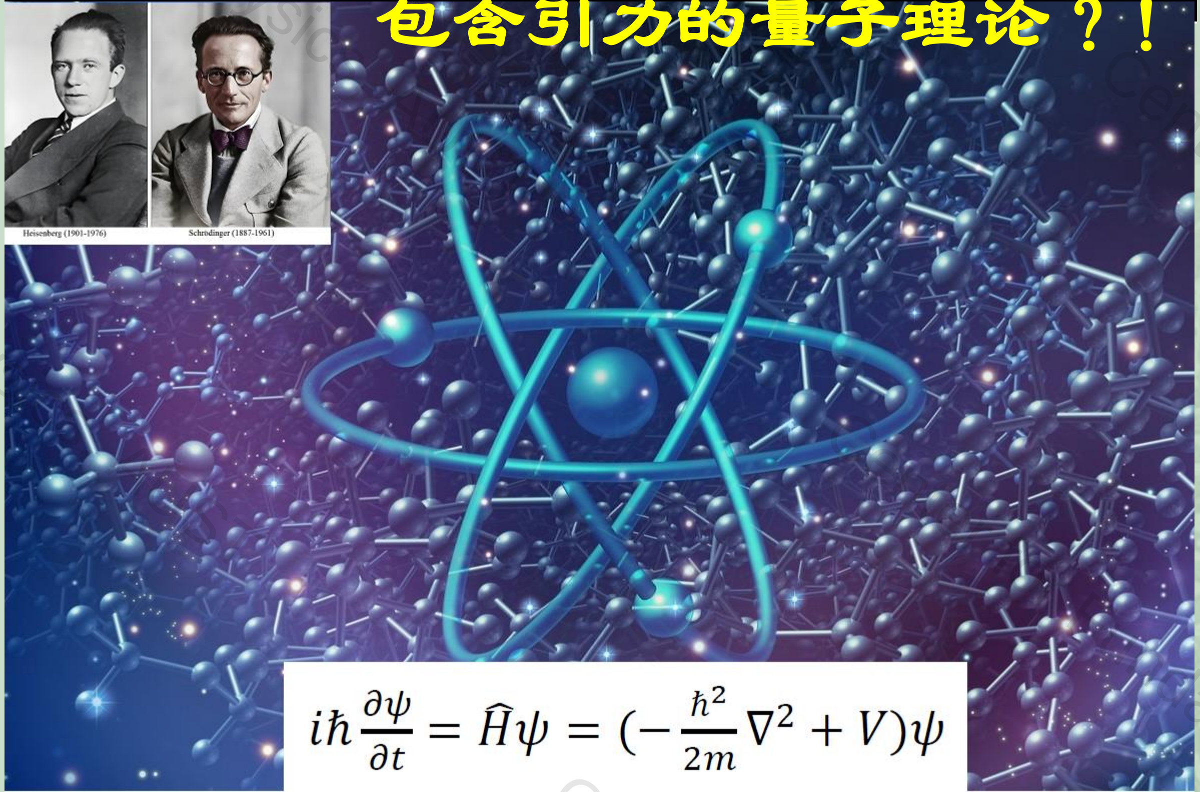
- Quantization（量子化）
- Symmetry（对称性）
- Phase Factor（相位因子）



$$F = G\frac{m_1 m_2}{r^2}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

包含引力的量子理论？！



$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi$$

空间量子化

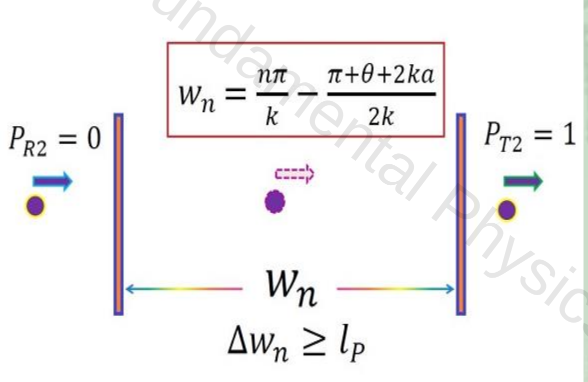
广义不确定原理 (GUP)

$$\boxed{\Delta x \gtrsim \frac{\hbar}{2\Delta p} + \alpha\frac{G\Delta p}{c^3}}$$

⇩

$$\boxed{l_{min} \sim l_P}$$

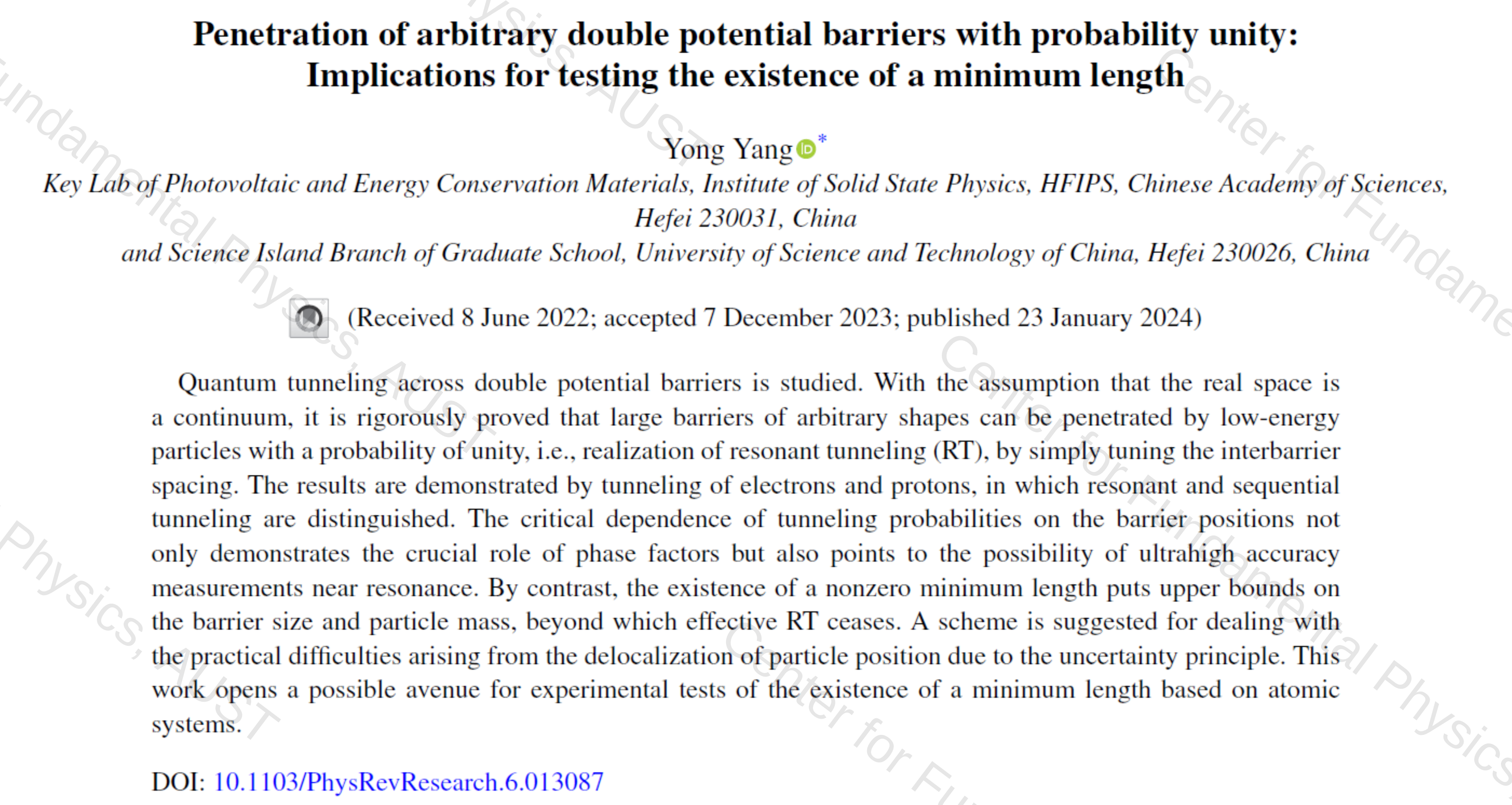
↕



$$\boxed{W_n = \frac{n\pi}{k} - \frac{\pi+\theta+2ka}{2k}}$$

$$\Delta w_n \geq l_P$$

More Details of the Work



Penetration of arbitrary double potential barriers with probability unity: Implications for testing the existence of a minimum length

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Quantum tunneling across double potential barriers is studied. With the assumption that the real space is a continuum, it is rigorously proved that large barriers of arbitrary shapes can be penetrated by low-energy particles with a probability of unity, i.e., realization of resonant tunneling (RT), by simply tuning the interbarrier spacing. The results are demonstrated by tunneling of electrons and protons, in which resonant and sequential tunneling are distinguished. The critical dependence of tunneling probabilities on the barrier positions not only demonstrates the crucial role of phase factors but also points to the possibility of ultrahigh accuracy measurements near resonance. By contrast, the existence of a nonzero minimum length puts upper bounds on the barrier size and particle mass, beyond which effective RT ceases. A scheme is suggested for dealing with the practical difficulties arising from the delocalization of particle position due to the uncertainty principle. This work opens a possible avenue for experimental tests of the existence of a minimum length based on atomic systems.

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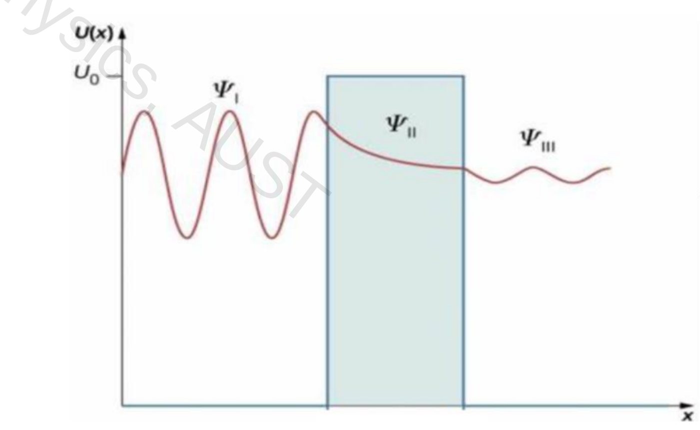
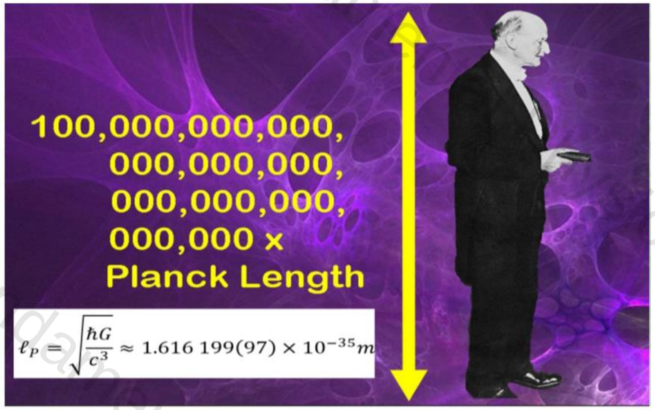
小　结

- 对量子隧穿的研究做了简要回顾
- 基于转移矩阵方法，得到了任意形状双势垒共振隧穿的一个定理，并应用于研究电子和质子的共振隧穿。
- 计算表明，共振隧穿对势垒的间距、粒子的能量单色性非常敏感
- 如果实空间是连续的，则总可以通过调节势垒间距实现共振隧穿
- 如果存在一个非零的最小空间尺度，则发生共振隧穿的势垒高度、宽度以及粒子质量等物理参数存在一个上限。



Resonant Tunneling

$$\frac{\hbar}{2\sqrt{2mE}}\sqrt{\frac{1}{R(1+R)}} \geq l_P$$



基础和应用两方面的意义

- **基础方面**

1. 再次证明微观粒子波函数相位的重要性
2. 通过检验最小空间尺度存在性，在量子物理和可能的量子引力理论之间建立起内在联系
3. 检验量子力学在多大/多小尺度上适用

- **应用方面**

1. 基于双势垒共振隧穿的超高精度测量
2. 量子阱结构束缚态能级的人工调控
3. 基于共振隧穿的同位素分离提纯

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